

Name: KEY

24

IB Mathematics SL Year 1

10 Quest RETAKE

2012

Calculator Section

30 Minutes

**Remember to check if you mode is in radians or
degrees!!!!**

1. Let $f(x) = \sin 2x$ and $g(x) = \sin (0.5x)$.

(a) Write down

(i) the minimum value of the function f ;

-1

(ii) the period of the function g .

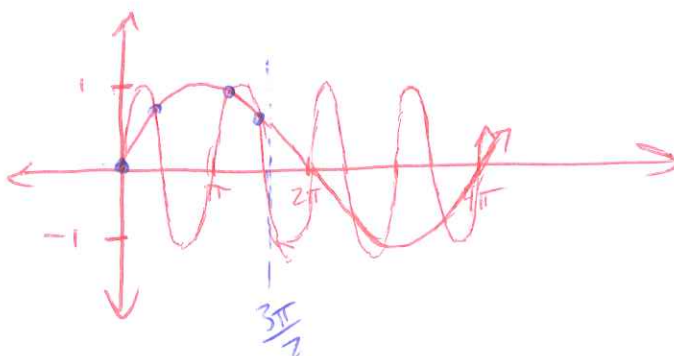
$$P = \frac{2\pi}{B} = \frac{2\pi}{0.5} = 4\pi$$

or 720°

(b) Consider the equation $f(x) = g(x)$.

SKETCH a graph and find the number of solutions to this equation, for $0 \leq x \leq \frac{3\pi}{2}$.

(Total 6 marks)



4 solutions

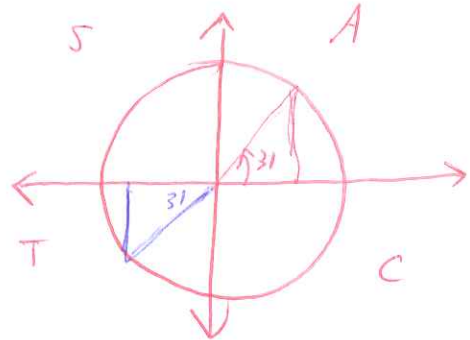
2. Solve the equation $\tan x = \frac{3}{5}$, for x in the interval $0^\circ \leq x \leq 360^\circ$, giving your answers to the nearest degree.

$$\tan^{-1}\left(\frac{3}{5}\right) = x$$

$$x \approx 30.96 \dots$$

$$x \approx 31^\circ$$

OR $180^\circ + 31^\circ = 211^\circ$



(Total 4 marks)

3. (a) Express $2 \cos^2 x + \sin x$ in terms of $\sin x$ only.

→ use ~~cos^2 x + sin^2 x = 1~~ $\cos^2 x + \sin^2 x = 1$
or $\cos^2 x = 1 - \sin^2 x$

$$\begin{aligned} & 2 \cos^2 x + \sin x \\ &= 2(1 - \sin^2 x) + \sin x \\ &= 2 - 2 \sin^2 x + \sin x \\ &= -2 \sin^2 x + \sin x + 2 \end{aligned}$$

- (b) Solve the equation $2 \cos^2 x + \sin x = 2$ for x in the interval $0 \leq x \leq \pi$, giving your answers exactly.

(Total 4 marks)

OR USE
GDC

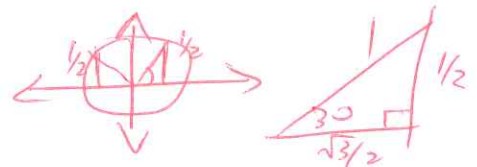
$$\begin{aligned} & -2 \sin^2 x + \sin x + 2 = 2 \\ & -2 \sin^2 x + \sin x = 0 \\ & \sin x (-2 \sin x + 1) = 0 \end{aligned}$$

$\therefore \sin x = 0$
when $x = 0$ or π
3.14

or $-2 \sin x + 1 = 0$
 $\sin x = \frac{1}{2}$

when
 $x = 30^\circ$ or 150°

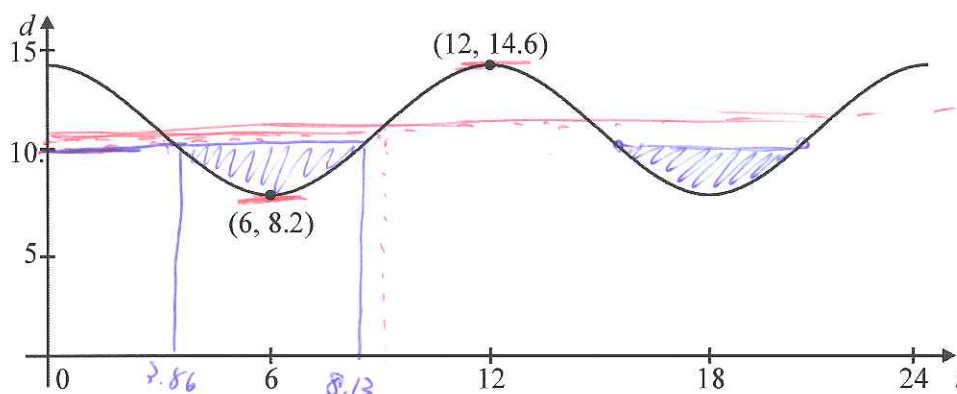
$\therefore x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$
0.52 2.62



4. A formula for the depth d metres of water in a harbour at a time t hours after midnight is

$$d = P + Q \cos\left(\frac{\pi}{6}t\right), \quad 0 \leq t \leq 24,$$

where P and Q are positive constants. In the following graph the point $(6, 8.2)$ is a minimum point and the point $(12, 14.6)$ is a maximum point.



- (a) Find the value of

(i) Q : $= \text{amplitude} = \frac{\text{Max} - \text{min}}{2} = \frac{14.6 - 8.2}{2} = 3.2$

(ii) P : $= \text{vert shift} = \frac{\text{Max} + \text{min}}{2} = \frac{14.6 + 8.2}{2} = 11.4$ (3)

- (b) Find the first time in the 24-hour period when the depth of the water is 10 metres.

$$\begin{aligned} y_1 &= 11.4 + 3.2 \cos\left(\frac{\pi}{6}t\right) \\ y_2 &= 10 \end{aligned} \quad \left. \vphantom{\begin{aligned} y_1 &= 11.4 + 3.2 \cos\left(\frac{\pi}{6}t\right) \\ y_2 &= 10 \end{aligned}} \right\} \text{intersect}$$

$\therefore t \approx 3.86$ (3 SF) hours

- (c) (i) Use the symmetry of the graph to find the **next** time when the depth of the water is 10 metres.

$$t \approx 8.13 \text{ hours}$$

- (ii) Hence find the time intervals in the 24-hour period during which the water is less than 10 metres deep.

$$\begin{aligned} \therefore 3.86 \leq t \leq 8.13 & \quad \text{or} \quad 3.86 + 12 \leq t \leq 8.13 + 12 \\ \text{hours} & \quad \quad \quad 15.86 \leq t \leq 20.13 \\ & \quad \quad \quad 15.9 \leq t \leq 20.1 \quad \text{3 SF} \quad \text{hours} \end{aligned} \quad (4)$$