

KEY

Name: _____

IB Mathematics SL Year 1

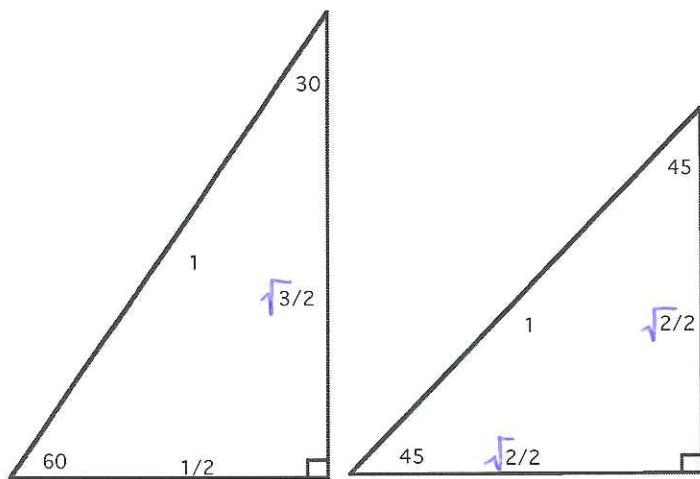
10 Quest RETAKE

Non-Calculator Section

14 Marks

25 Minutes

Special Triangles:



$$\text{Note: } \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \left(\frac{\sqrt{2}}{\sqrt{2}} \right) = \frac{\sqrt{2}}{2}$$

xxxxxxxxxxxx

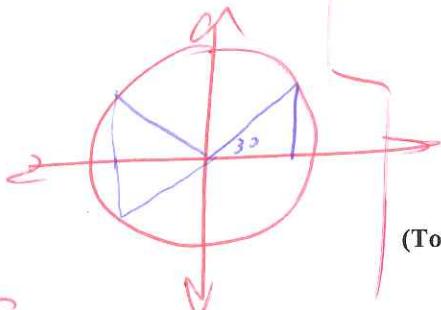
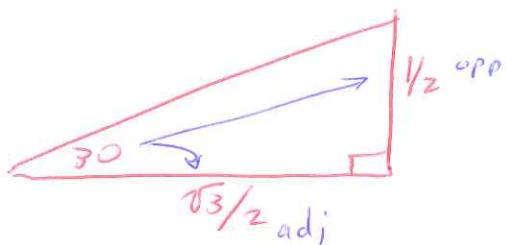
Make sure to write this in!

1. Solve the equation $3 \sin^2 x = \cos^2 x$, for $0^\circ \leq x \leq 180^\circ$. (Hint: $\frac{\sin x}{\cos x} = \tan x$)

$$\frac{3 \sin^2 x}{\cos^2 x} = 1$$

$$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$$

$$\tan x = \frac{1}{\sqrt{3}} = \frac{1/2}{\sqrt{3}/2} \leftarrow \begin{matrix} \text{opp} \\ \text{adj} \end{matrix}$$

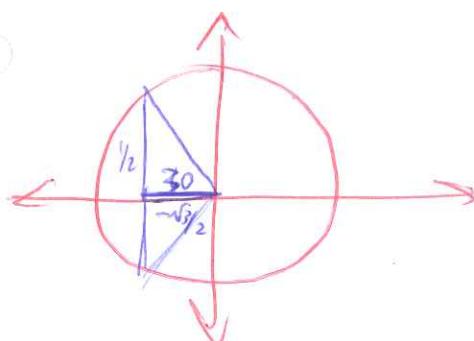


(Total 4 marks)

$$\therefore x = 30^\circ \text{ (only solution on } 0 \leq x \leq 180^\circ)$$

2. Find exact solutions for $2 \cos x + \sqrt{3} = 0$, $0 \leq x \leq 3\pi$. $\leftarrow \frac{18\pi}{6}$

$$\cos x = -\frac{\sqrt{3}}{2}$$



$$\therefore x = \begin{cases} 150^\circ \\ 210^\circ \end{cases} + 2k\pi$$

$$= \begin{cases} 5\pi/6 \\ 7\pi/6 \end{cases} + 2k\pi$$

check k values

$$k=0$$

$$5\pi/6, 7\pi/6$$

$$150^\circ, 210^\circ$$

$$\begin{aligned} k &= 1 \\ &= 5\pi/6 + 2\pi \quad \text{or} \quad 7\pi/6 + 2\pi \\ &= 5\pi/6 + \frac{12\pi}{6} \\ &= 17\pi/6 \\ &= 19\pi/6 \dots \text{too big} \end{aligned}$$

(Total 4 marks)

3. Given that $\sin x = \frac{1}{3}$, where x is an acute angle, find the exact value of

(a) $\cos x$; $\sin^2 x + \cos^2 x = 1$
 $\left(\frac{1}{3}\right)^2 + \cos^2 x = 1$
 $\cos^2 x = 1 - \frac{1}{9}$

(b) $\cos 2x$. $\cos^2 x = \frac{9}{9} - \frac{1}{9}$

(Total 6 marks)

$$\begin{aligned}\cos^2 x &= \frac{8}{9} \\ \therefore \cos x &= \pm \sqrt{\frac{8}{9}} \\ \text{choose } \cos x &= \sqrt{\frac{8}{9}} \\ &= \sqrt{\frac{8}{9}} \\ &= \sqrt{\frac{4 \cdot 2}{9}} \\ &= 2\sqrt{\frac{2}{3}}\end{aligned}$$

$$\begin{aligned}\therefore \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ \cos 2x &= \frac{8}{9} - \frac{1}{9} \\ &= \frac{7}{9}\end{aligned}$$

EXTRA CREDIT :

Show that $\frac{\sin 2\alpha - \sin \alpha}{\cos 2\alpha - \cos \alpha + 1}$ simplifies to $\tan \alpha$.

$$\sin 2\alpha = 2\sin \alpha \cos \alpha$$

$$= \frac{2\sin \alpha \cos \alpha - \sin \alpha}{\cos 2\alpha - \cos \alpha + 1}$$

$$= \frac{\sin \alpha (2\cos \alpha - 1)}{\cos 2\alpha - \cos \alpha + 1}$$

$$= \frac{\sin \alpha (2\cos \alpha - 1)}{(2\cos^2 \alpha - 1) - \cos \alpha + 1}$$

$$= \frac{\sin \alpha (2\cos \alpha - 1)}{2\cos^2 \alpha - \cos \alpha}$$

$$= \frac{\sin \alpha (2\cos \alpha - 1)}{\cos \alpha (2\cos \alpha - 1)}$$

$$= \frac{\sin \alpha}{\cos \alpha}$$

$$= \tan \alpha$$

$$\begin{aligned} &\downarrow \text{Use } \cos 2\alpha \\ &= 2\cos^2 \alpha - 1 \end{aligned}$$