Name: KEY

Note:
E.C. was +6 & directly added on to overall %age.

IB Mathematics SL Year 1

Advanced Trigonometry Ch. 10 Quest

February …… 2012

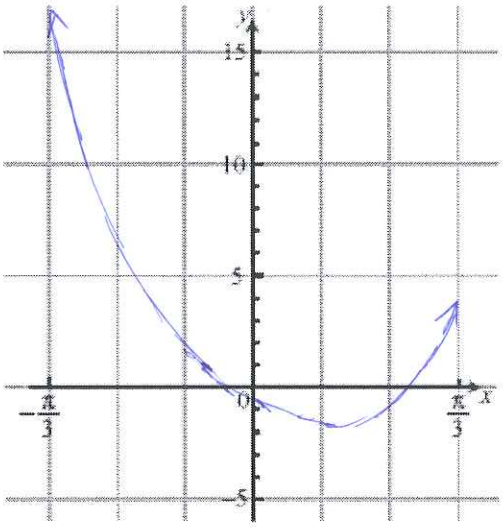
Calculator Section

15 Minutes

MODE ⇒ Radians!

1. Let $f(x) = 4\tan^2 x - 4\sin x$, $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$.

(a) On the grid below, sketch the graph of $y = f(x)$.



(3)

(b) Solve the equation $f(x) = 1$.

(3)

$x \approx -0.2072\ldots$

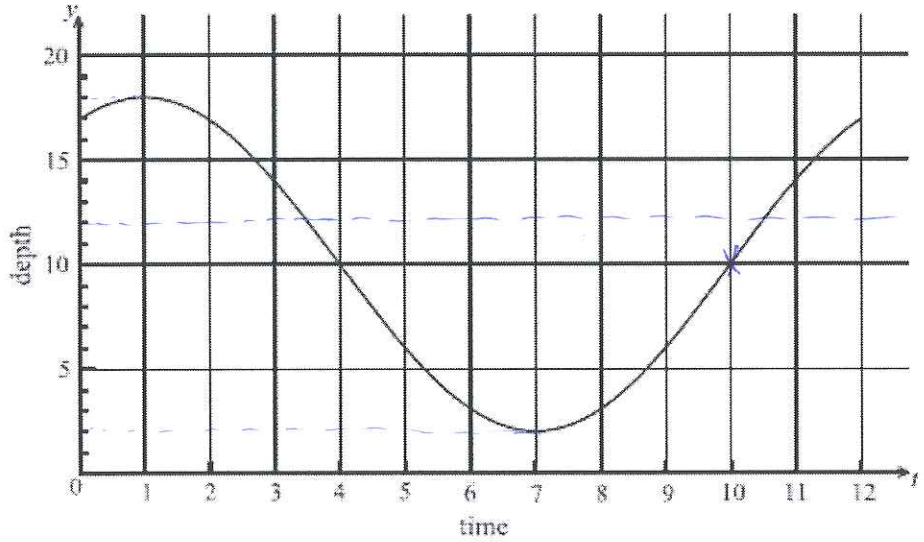
≈ -0.207 (3SF)

or $x \approx 0.7719\ldots$

≈ 0.772 (3SF)

(Total 6 marks)

3. The following graph shows the depth of water, y metres, at a point P, during one day. The time t is given in hours, from midnight to noon.



(a) Use the graph to write down an estimate of the value of t when

(i) the depth of water is minimum;

7 hours ∴ 7am

(ii) the depth of water is maximum;

1 or 13 ∴ 1 am

(iii) the depth of the water is increasing most rapidly.

10am

(3)

(b) The depth of water can be modelled by the function $y = A\cos(B(t-1)) + F$.

(i) Show that $A = 8$. $\frac{18-2}{2} = \frac{16}{2} = 8$

(ii) Write down the value of F.

10

(iii) Find the value of B.

$P = \frac{2\pi}{B}$ $12 = \frac{2\pi}{B}$ $B = \frac{\pi}{6}$

(6)

(c) A sailor knows that he cannot sail past P when the depth of the water is less than 12 m. Calculate the values of t between which he cannot sail past P.

(2)
(Total 11 marks)

$12 = 8\cos\left(\frac{\pi}{6}(x-1)\right) + 10$

$3.52 \le t \le 10.48\ldots$

3.52 hours $\le t \le$ 10.5 hours

Notes :

Extra Credit

"Show" that

$$\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = \frac{2}{\cos\theta}$$

$$\left(\frac{\cos\theta}{\cos\theta}\right)\left(\frac{\cos\theta}{1+\sin\theta}\right) + \left(\frac{1+\sin\theta}{\cos\theta}\right)\left(\frac{1+\sin\theta}{1+\sin\theta}\right) \quad \text{(HINT OF)}$$

$$= \frac{\cos^2\theta}{\cos\theta + \sin\theta\cos\theta} + \frac{1 + 2\sin\theta + \sin^2\theta}{\cos\theta + \sin\theta\cos\theta}$$

$$= \frac{\cancel{\cos^2\theta} + 1 + 2\sin\theta + \cancel{\sin^2\theta} + 1}{\cos\theta + \sin\theta\cos\theta}$$

$$= \frac{2 + 2\sin\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{2\cancel{(1+\sin\theta)}}{\cos\theta\cancel{(1+\sin\theta)}}$$

$$= \frac{2}{\cos\theta}$$