

Name: KEY

IB Mathematics SL Year 1

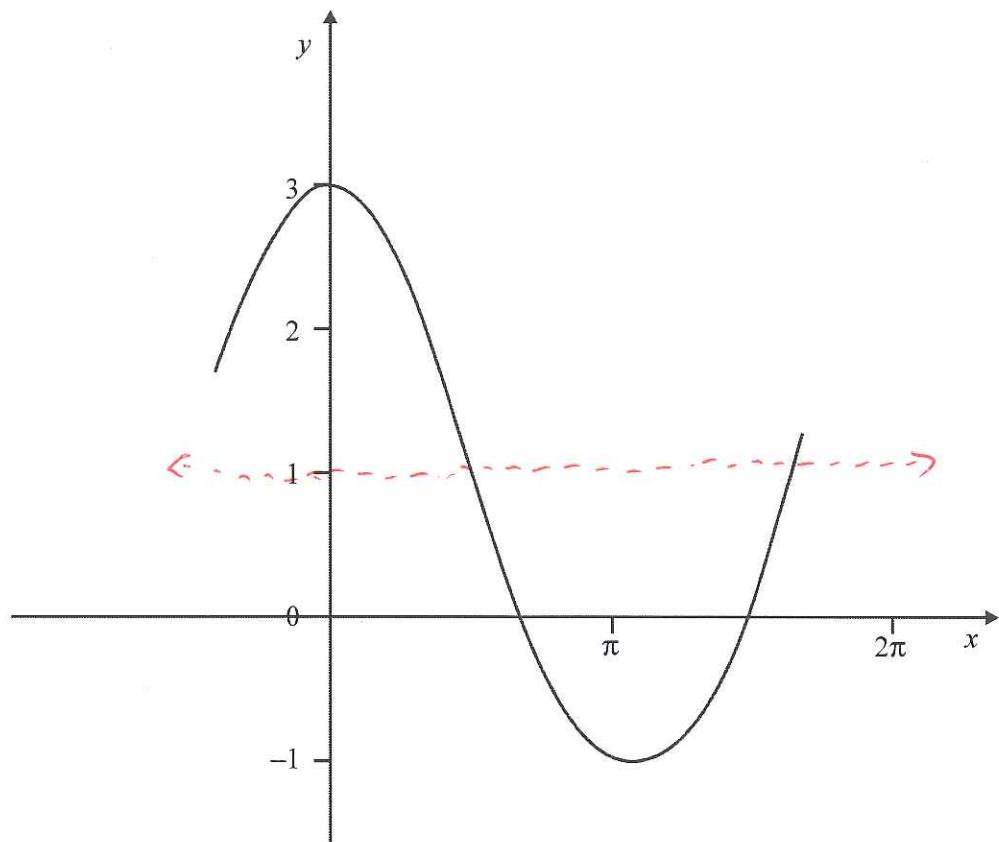
Ch. 10 Advanced Trigonometry Quest

February 2012

Non-Calculator Section

25 Minutes

1. Part of the graph of $y = p + q \cos x$ is shown below. The graph passes through the points $(0, 3)$ and $(\pi, -1)$.



Find the value of

(a) p ; $y = q \cos x + p$ $\downarrow (0, 3)$ $y = q \cos x + p$ $\downarrow (\pi, -1)$

$$3 = q \cos(0) + p$$

$$-1 = q \cos(\pi) + p$$

$$-1 = -q + p$$

max/min \checkmark

(1) $\begin{cases} 3 = q + p \\ -1 = -q + p \end{cases}$ \downarrow

(2) $\begin{cases} 3 = q + p \\ -1 = -q + p \end{cases}$ $\therefore p = 1$

(b) q . $2 = 2p$

$$p = 1$$

(Total 6 marks)

max-min \checkmark

$$q = 3 - p$$

$$q = 3 - 1$$

$$q = 2$$

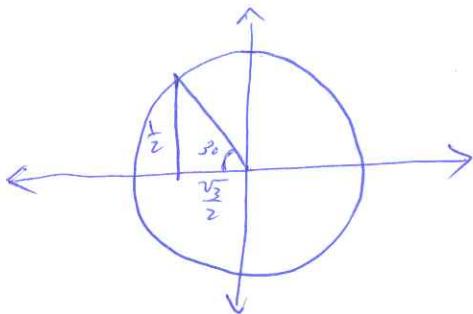
(from looking @ amplitude)

(or any other
method to solve)

$$90^\circ < \theta < 180^\circ$$

2. Given that $\sin \theta = \frac{1}{2}$, and $270^\circ < \theta < 360^\circ$,

(a) find the value of $\cos \theta$; (hint: Use the unit circle to help you)



(b) find the value of θ .

$$\therefore \theta = 150^\circ$$

$$\therefore \cos \theta = -\frac{\sqrt{3}}{2}$$

$$\text{alt solution} \quad \sin^2 \theta + \cos^2 \theta = 1$$

$$\frac{1^2}{2} + \cos^2 \theta = 1$$

$$\cos^2 \theta = \frac{4}{4} = \frac{1}{4}$$

$$\cos^2 \theta = \frac{1}{4}$$

$$\cos \theta = \pm \frac{\sqrt{3}}{2} \quad \text{choose } (-) \text{ soln}$$

(1)

(1)

(b) write down the exact value of $\tan \theta$.

(Total 4 marks)

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$= \frac{1/2}{-\sqrt{3}/2}$$

$$= -\frac{1}{\sqrt{3}}$$

$$= -\frac{\sqrt{3}}{3}$$

(1)

3. Consider the trigonometric equation $2 \sin^2 x = 1 + \cos x$.

- (a) Write this equation in the form $f(x) = 0$, where $f(x) = a \cos^2 x + b \cos x + c$, and $a, b, c \in \mathbb{Z}$.

$$2(1 - \cos^2 x) = 1 + \cos x$$

$$2 - 2\cos^2 x = 1 + \cos x$$

$$0 = 2\cos^2 x + \cos x - 1$$

Note: $1 - 2\sin^2 x$
 $= 2\cos^2 x - 1$ also!

- (b) Factorize $f(x)$.

$$0 = (2\cos x - 1)(\cos x + 1)$$

- (c) Solve $f(x) = 0$ for $0 \leq x \leq 2\pi$.

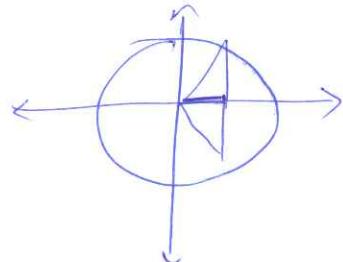
$$\therefore \cos x + 1 = 0 \quad \text{or} \quad 2\cos x - 1 = 0$$

$$\cos x = -1$$

$$\cos x = \frac{1}{2}$$

when $x = \pi$

when



$$x = 60^\circ \quad \text{or} \quad 300^\circ$$

$$x = \frac{\pi}{3} \quad \text{or} \quad \frac{5\pi}{3}$$

4. Consider $y = \sin\left(x + \frac{\pi}{9}\right)$.

- (a) The graph of y intersects the x -axis at point A. Find the x -coordinate of A, where $0 \leq x \leq \pi$. (hint: set $y = 0$ to find x -intercept)

$$0 = \sin\left(x + \frac{\pi}{9}\right)$$

$$\therefore \sin^{-1}(0) = x + \frac{\pi}{9}$$

$$\therefore x + \frac{\pi}{9} = \begin{cases} 0 \\ \pi \end{cases} + k \cdot 2\pi$$

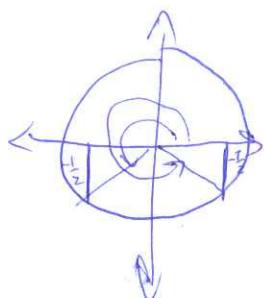
$$x = \begin{cases} -\frac{\pi}{9} \\ \pi - \frac{\pi}{9} \end{cases} + k \cdot 2\pi$$

(b) Solve the equation $\sin\left(x + \frac{\pi}{9}\right) = -\frac{1}{2}$, for $0 \leq x \leq 2\pi$.

$$\sin\left(-\frac{1}{2}\right) = x + \frac{\pi}{9}$$

$$x = \begin{cases} -\frac{\pi}{9} \\ \frac{7\pi}{6} \end{cases} + k \cdot 2\pi$$

\therefore ONLY choose $\frac{7\pi}{9}$



(Total 6 marks)

$$\therefore 210^\circ \text{ or } 330^\circ$$

$$\therefore x + \frac{\pi}{9} = \begin{cases} \frac{7\pi}{6} \\ \frac{11\pi}{6} \end{cases} + k \cdot 2\pi$$

$$0 \leq x \leq \frac{36\pi}{18}$$

when $k=0$

when $k=1$

$$X = \frac{7\pi}{6}, \frac{11\pi}{6}$$

too big

$$= \frac{7\pi}{6} - \frac{\pi}{9} \quad \left| \quad \frac{11\pi}{6} - \frac{\pi}{9} \right.$$

$$= \frac{21\pi}{18} - \frac{2\pi}{18} \quad \left| \quad = \frac{33\pi}{18} - \frac{2\pi}{18} \right.$$

$$= \frac{19\pi}{18} \quad \left| \quad = \frac{31\pi}{18} \right.$$