Chapter 13:
Section B:

APPLICATIONS OF VECTOR LINES. Notes and examples:

(Have sketch pad grid also)

1. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + t\begin{pmatrix} -6 \\ -8 \end{pmatrix}$ is the vector equation of the path of a particle.

t is the time in seconds, $t \geq 0$. The distance is measured in units of meters.

a.) **Find** the particles *initial position.*

When $t=0$, position vector $= \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ or

coordinates $\rightarrow (3,4)$

b.) **Find** the *velocity vector* of the particle.

$\begin{pmatrix} -6 \\ -8 \end{pmatrix}$

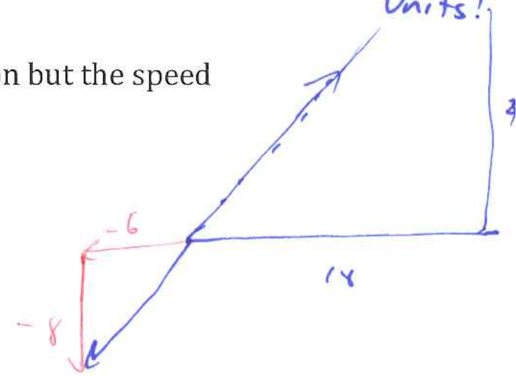
c.) **Find** the *speed* of the particle.

Speed $= \left|\begin{pmatrix} -6 \\ -8 \end{pmatrix}\right| = \sqrt{(-6)^2+(-8)^2} = \sqrt{36+64} = \sqrt{100} = 10\boxed{m/s}$

units!

d.) If the particle continues in the **opposite direction** direction but the speed increases to 30m/s, state the new *velocity vector.*

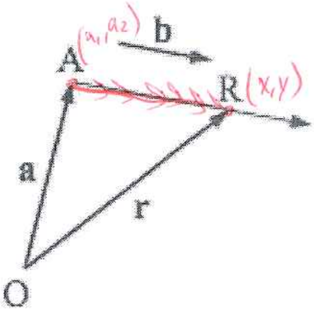
$-3\begin{pmatrix} -6 \\ -8 \end{pmatrix} = \begin{pmatrix} 18 \\ 24 \end{pmatrix}$



CONSTANT VELOCITY PROBLEMS

If a body has initial position vector **a** and moves with constant velocity **b**, its position at time t is given by

$$\mathbf{r} = \mathbf{a} + t\mathbf{b}, \quad \text{for} \quad t \geqslant 0.$$



2. An object is **initially** at (2,6) and moves with **velocity vector** $2\hat{i}-\hat{j}$ ***Find:***

a.) the position of the object at any time t (t is in minutes)

$$\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

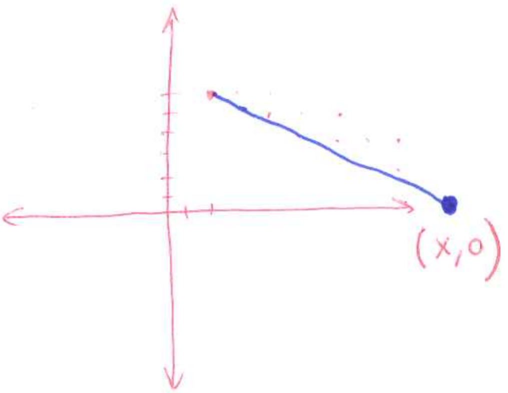
$$\therefore \begin{cases} x = 2 + 2t \\ y = 6 - t \end{cases}$$

$\longrightarrow$ position $(x, y) \longrightarrow (2+2t,\ 6-t)$ for any t.

b.) the position after 5 minutes. $t = 5$

$$\therefore \ (2 + 2(5),\ 6 - 5)$$
$$= (12, 1)$$

c.) The time when the object is due east of the origin.



$\therefore \ y = 0$

$\therefore \ 6 - t = 0$

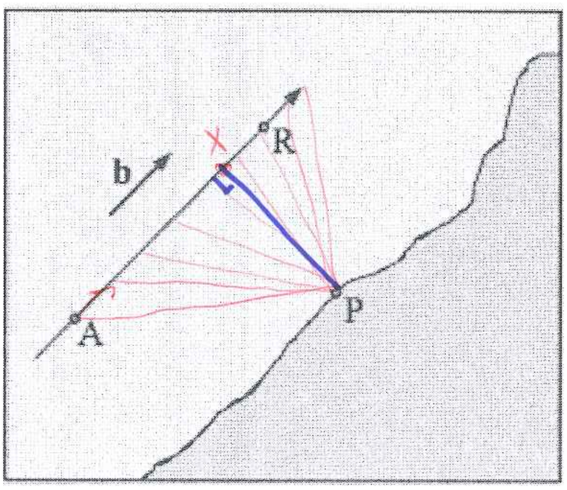
$\therefore \ t = 6$

Note: if $t = 6$

then $x = 2 + 2(6)$

$x = 14.$

THE CLOSEST DISTANCE FROM A POINT TO A LINE

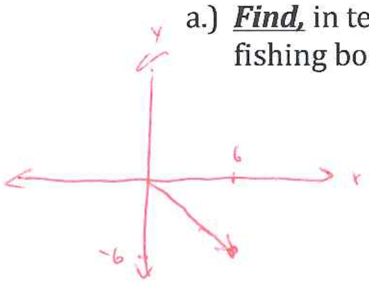


$\longrightarrow$ closest when $\perp$ perpendicular!

$\therefore \ \overrightarrow{AR} \bullet \overrightarrow{PX} = 0$

3. An ocean liner is at (6,-6), cruising at 10km/h in the direction $\begin{pmatrix}-3\\4\end{pmatrix}$. A fishing boat is anchored at (0,0) (Distances in km)

a.) ***Find,*** in terms of **i** and **j**, the "O.G." position vector of the liner from the fishing boat.



$\therefore \begin{pmatrix}6\\-6\end{pmatrix}$ or $6\hat{i} - 6\hat{j}$

b.) Write an expression for the position vector of the liner at any time t hours after it has sailed from (6, -6).

$\vec{r} = \vec{a} + t\vec{b}$

point

Velocity vector (speed = 10km/h)

$\left|\begin{pmatrix}-3\\4\end{pmatrix}\right| = \sqrt{(-3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$\therefore$ Velocity vector must be $2\begin{pmatrix}-3\\4\end{pmatrix} = \begin{pmatrix}-6\\8\end{pmatrix}$

$\therefore$ Position $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}6\\-6\end{pmatrix} + t\begin{pmatrix}-6\\8\end{pmatrix}$

c.) ***Find*** when the liner is due east of the fishing boat.

Vector $\rightarrow \begin{pmatrix}x = 6 - 6t\\y = -6 + 8t\end{pmatrix}$

$\therefore y = 0 = -6 + 8t$

$6 = 8t$

$t = \frac{6}{8}$

$t = 3/4 = 0.75$ hours

d.) ***Find*** the time and position of the liner when it is nearest to the fishing boat.

$\boxed{\text{sketch graph (Autograph)}}$

Nearest when perpendicular!!

$\therefore \begin{pmatrix}-3\\4\end{pmatrix} \cdot \begin{pmatrix}(6-6t)-0\\(-6+8t)-0\end{pmatrix} = 0$

$\begin{pmatrix}-3\\4\end{pmatrix} \cdot \begin{pmatrix}6-6t\\-6+8t\end{pmatrix} = 0$

$-3(6-6t) + 4(-6+8t) = 0$

$-18 + 18t - 24 + 32t = 0$

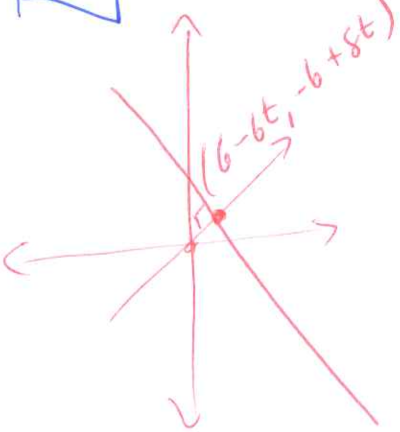
$-42 + 50t = 0$

$50t = 42$

$t = 42/50 = 0.84$ hours

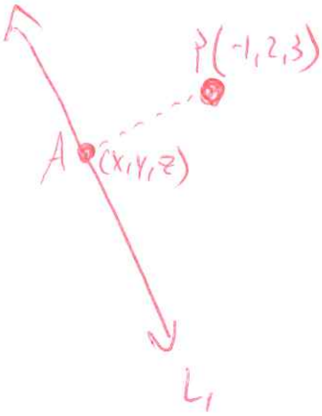
$\rightarrow P(6 - 6(0.84), -6 + 8(0.84))$

$P(0.96, 0.72)$



4. Consider the point P(-1, 2, 3) and the line with parametric equations
$x=1+2t$, $y=-4+3t$, $z=3+t$.

a.) ***Find*** the coordinates of the foot of the perpendicular from P to the line.



$$\begin{cases} x = 1+2t \\ y = -4+3t \\ z = 3+t \end{cases}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

$\therefore \overrightarrow{PA} \bullet \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} x - -1 \\ y - 2 \\ z - 3 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = 0$

$= \begin{pmatrix} (1+2t)+1 \\ (-4+3t)-2 \\ (3+t)-3 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = 0$

$= \begin{pmatrix} 2+2t \\ -6+3t \\ t \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = 0$

$2(2+2t)+3(-6+3t)+t=0$

$4+4t \quad -18+9t+t=0$

$-14+14t=0$

$\therefore t=1$

$\therefore x = 1+2(1) = 3$

$y = -4+3(1) = -1$

$z = 3+1 = 4$

or $(3,-1,4)$

b.) ***HENCE,*** find the shortest distance from P to the line.

$\therefore \left|\overrightarrow{PA}\right| = \text{size}$

$= \left|\begin{pmatrix} 2+2(1) \\ -6+3(1) \\ 1 \end{pmatrix}\right|$

$= \left|\begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}\right|$

$= \sqrt{4^2+(-3)^2+1^2}$

$= \sqrt{16+9+1}$

$= \sqrt{26}$ units

5.

The line L_1 passes through the points $P(2, 4, 8)$ and $Q(4, 5, 4)$.

(a) (i) Find $\vec{PQ}$.

(ii) Hence write down a vector equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + s\boldsymbol{b}$. [4 marks]

The line L_2 is perpendicular to L_1, and parallel to $\begin{pmatrix} 3p \\ 2p \\ 4 \end{pmatrix}$, where $p \in \mathbb{Z}$.

(b) (i) Find the value of p.

(ii) Given that L_2 passes through $R(10, 6, -40)$, write down a vector equation for L_2. [7 marks]

(c) The lines L_1 and L_2 intersect at the point A. Find the x-coordinate of A. [7 marks]

a) i) $\vec{PQ} = \begin{pmatrix} 4-2 \\ 5-4 \\ 4-8 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$

ii) $L_1 : \vec{r} = \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$

b) (i) If $L_2 \perp L_1$, then

$$\begin{pmatrix} 3p \\ 2p \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} = 0$$

$$6p + 2p - 16 = 0$$

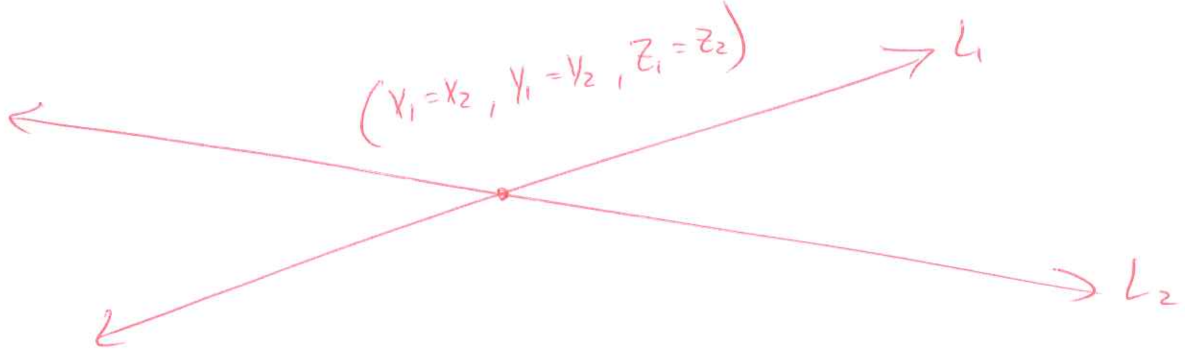
$$8p = 16$$

$$p = 2$$

ii) $\therefore L_2 : \vec{r} = \begin{pmatrix} 10 \\ 6 \\ -40 \end{pmatrix} + s\begin{pmatrix} 3\cdot 2 \\ 2\cdot 2 \\ 4 \end{pmatrix}$

$$\vec{r} = \begin{pmatrix} 10 \\ 6 \\ -40 \end{pmatrix} + s\begin{pmatrix} 6 \\ 4 \\ 4 \end{pmatrix}$$

c) $L_1: \begin{cases} x_1 = 2 + 2t \\ y_1 = 4 + t \\ z_1 = 8 - 4t \end{cases}$ $L_2: \begin{cases} x_2 = 10 + 6s \\ y_2 = 6 + 4s \\ z_2 = -40 + 4s \end{cases}$



$$\therefore \begin{cases} x = 2 + 2t = 10 + 6s \\ y = 4 + t = 6 + 4s \end{cases}$$
$$z = 8 - 4t = -40 + 4s$$

2eq : 2 unknowns :

$2 + 2t = 10 + 6s \rightarrow \begin{cases} 2t - 6s = 8 \\ + (t - 4s = 2) \cdot -2 \end{cases}$

$4 + t = 6 + 4s \rightarrow$

$$\begin{array}{r} 2t - 6s = 8 \\ + \; -2t + 8s = -4 \\ \hline +2s = 4 \end{array}$$

$\therefore s = +2$

$\therefore t - 4(+2) = 2$

$t - 8 = 2$

$t = +10$

$\therefore x = 2 + 2(10) = 22$

or $x = 10 + 6(2) = 22$ ✓