15H Binomial Probabilities

Binomial expansion review:

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^2$

$T_1 \quad T_2 \quad T_3 \quad T_4$

$(a+b)^n$

Pascal's Δ

1
1 2 1
1 3 3 1
1 4 6 4 1
⋮ etc.

General term of binomial expansion

$(r+1)^{th}$ term

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

See F-booklet

$\therefore T_1 \Rightarrow (r=0), (n=3)$

$\therefore T_{0+1} = \binom{3}{0} a^{3-0} \cdot b^0$

$= 1 \cdot a^3 \cdot 1$

$= a^3$ ✓

or T_3

$= T_{2+1} = \binom{3}{2} a^{3-2} \cdot b^2$

$= 3a^1b^2$

$= 3ab^2$ ✓

See GDC

also $\binom{n}{0} = \binom{n}{n} = 1$.

i.e. $\binom{4}{2}$ $\{A, B, C, D\}$

$= 6$ $\{AB\}$

Also $\{AC\}$

^nC_r $\{AD\}$

$^4C_2$ $\{B, C\}$

⋮ $\{D, D\}$

order does not matter $\{C, D\}$

i.e. $\{A, B\} = \{B, A\}$

(how many ways can you shake hands in class room combinations vs permutations)

$\therefore$ From F-booklet

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n$$

$T_1 \quad T_2 \quad + \cdots + \quad T_{r+1} \quad + \cdots \quad T_{n+1}$

Ex: $\left(2x - \frac{1}{b}\right)^{14}$ Find T_8

let

$n = 14$

$a = 2x$

$b = -\frac{1}{b}$

$r + 1 = 8$

$\therefore r = 7.$

$$T_{r+1} = \binom{n}{r} a^{n-r} \cdot b^r$$

$$T_8 = \binom{14}{7}(2x)^{14-7} \cdot \left(-\frac{1}{b}\right)^7$$

$$= \binom{14}{7}(2x)^7 \cdot \left(-\frac{1}{b}\right)^7$$

$$= 3432(2x)^7\left(-\frac{1}{b}\right)^7$$

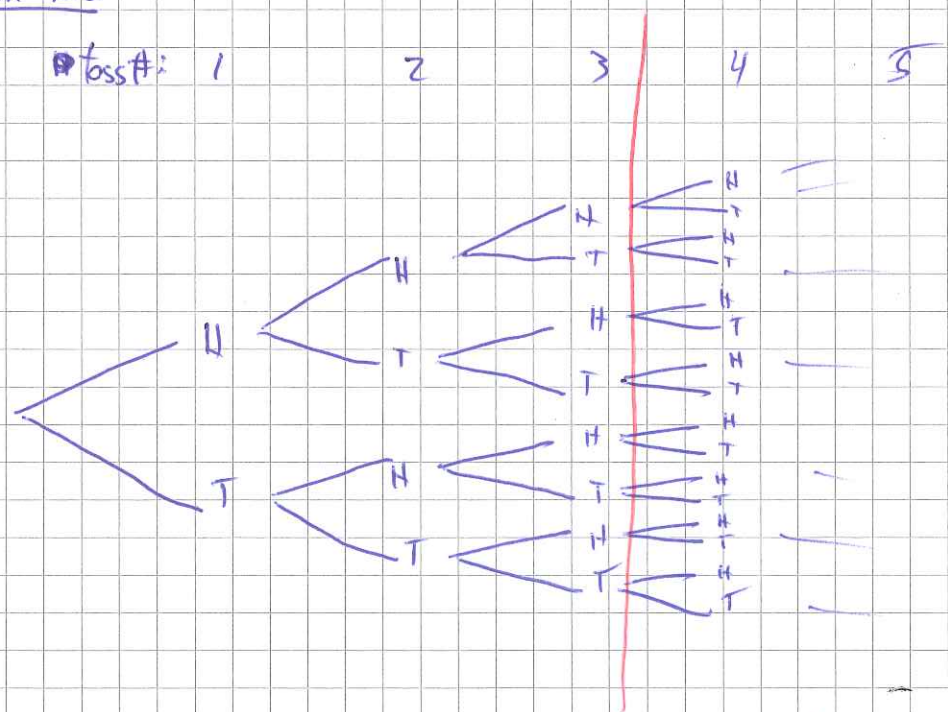
$\left(\begin{matrix} 14C7 \\ \text{GDC} \end{matrix}\right)$

In general for $(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$

(above would be $T_1 + T_2 + T_3 + T_4 + \cdots + T_{13} + T_{14}$.

Binomial expansion in probabilities

Coin toss:



anything past here start becoming hard to represent using tree diagr

See text book pg 444

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(14C7 GDC)

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(above would be $T_1 + T_2 + T_3 + T_4 + \cdots + T_{13} + T_{14}$.

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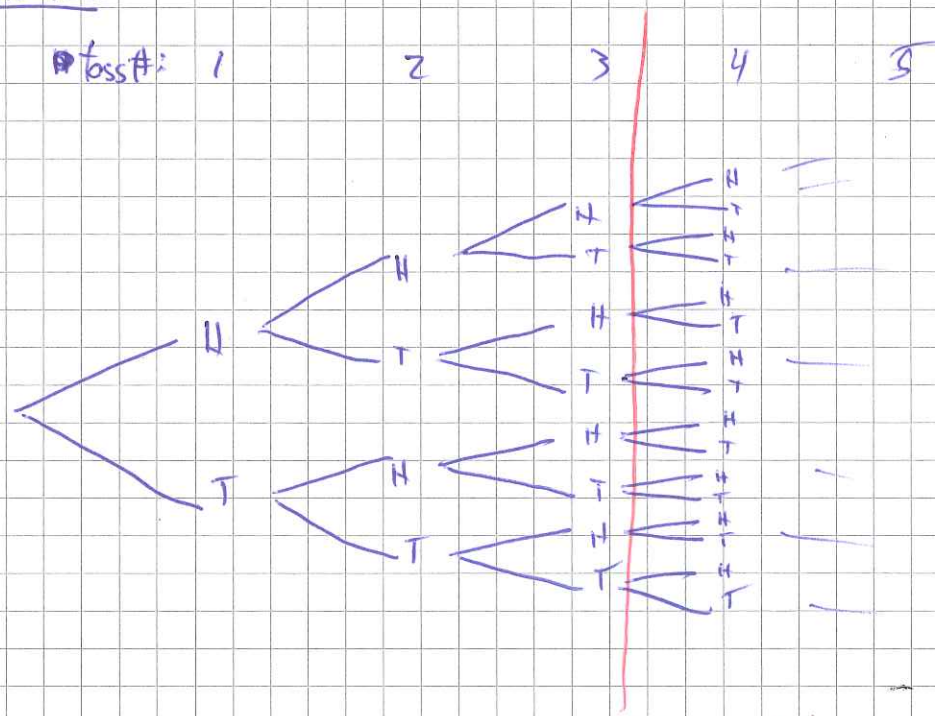

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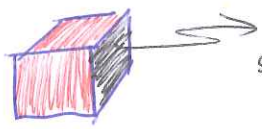
Write on the board with this projected.



rest of sides are black.

H BINOMIAL PROBABILITIES

Consider a die which has 2 red faces and 4 black faces. We roll the die three times and record the results.

If R represents "the result is red" and B represents "the result is black", the possible outcomes are as shown alongside:

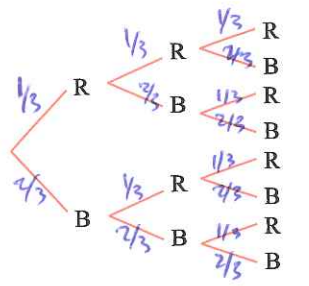
All red	*2 red 1 black*	*1 red 2 black*	*All black*
RRR	BRR	RBB	BBB
	RBR	BRB	
	RRB	BBR	
1	3	3	1

Notice that the ratio of possible outcomes is $1:3:3:1$.

Now for each die, $P(R) = \frac{1}{3}$ and $P(B) = \frac{2}{3}$.

So, for rolling the die 3 times we have the following events and probabilities:

Tosses ① ② ③



Event	*Outcome*	*Probabilities*	*Total Probability*
all red	RRR	$\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$	$\left(\frac{1}{3}\right)^3 = \frac{1}{27}$
2 red and 1 black	BRR	$\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$	$3\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right) = \frac{6}{27}$
	RBR	$\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)$	
	RRB	$\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)$	
1 red and 2 black	RBB	$\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)$	$3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 = \frac{12}{27}$
	BRB	$\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)$	
	BBR	$\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)$	
all black	BBB	$\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)$	$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$

Notice that $\left(\frac{1}{3}\right)^3 + 3\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right) + 3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3$ is the binomial expansion for $\left(\frac{1}{3} + \frac{2}{3}\right)^3$.

$= 1$

In general, If A is an event with probability p of occurring and its complement A' has probability $q = 1 - p$ of occurring, then the **probability generator** for the various outcomes over n **independent trials** is $(p+q)^n$.

For example:

Suppose A is the event of a randomly chosen light globe being faulty, with $P(A) = p = 0.03$ and $P(A') = q = 0.97$.

faulty not faulty

If four independent samples are taken, the probability generator is $(0.03 + 0.97)^4$

$$= (0.03)^4 + 4(0.03)^3(0.97) + 6(0.03)^2(0.97)^2 + 4(0.03)(0.97)^3 + (0.97)^4$$

4As 3As and 1A′ 2As and 2A′s 1A and 3A′s 4A′s

4-0 x=3 4-3 x=2 4-2 x=1 4-1 x=0 4-0

(free diagram gets confusing!)

Notice that $P(A \text{ occurs } x \text{ times } \textbf{ and } A' \text{ occurs } n - x \text{ times}) = \binom{n}{x} p^x q^{n-x}$.

Example: I hit the door 80% of the time. If I throw 5 markers, find probability generator and hence the chance of hitting the door

a) twice only

b) at most 3 times (vs. less than 3 times)

c) at least 4 times

Let H = hitting the door.

$\therefore P(H) = 0.80$
$P(H') = 0.20$

The probability generator is $(p+q)^n$

$= (0.80 + 0.20)^5$

See △

$= \binom{5}{0}(0.8)^5 + \binom{5}{1}(0.8)^4(0.20)^1 + \binom{5}{2}(0.8)^3(0.2)^2 + \binom{5}{3}(0.8)^2(0.2)^3 + \binom{5}{4}(0.8)^1(0.2)^4 + \binom{5}{5}(0.2)^5$

$= 1(0.8)^5 + 5(0.8)^4(0.2)^1 + 10(0.8)^3(0.2)^2 + 10(0.8)^2(0.2)^3 + 5(0.8)(0.2)^4 + (0.2)^5$

Probabilities: 5 hits + 0 misses; 4 hits + 1 miss; 3 hits + 2 misses; 2 hits + 3 misses; 1 hit + 4 misses; 0 hits + 5 misses

Let X = # of markers to hit the door.

a) $P(X=2) = 10(0.8)^2(0.2)^3$

$= 0.0512$

binompdf

b) $P(X = 3, 2, 1 \text{ or } 0) = P(X \le 3)$

$= 10(0.8)^3(0.2)^2 + 10(0.8)^2(0.2)^3 + 5(0.8)(0.2)^4 + (0.2)^5$

$= 0.26272$

binomcdf

c) $P(X = 4, 5) = P(X > 3) = 0.8^5 + 5(0.8)^4(0.2)$ OR $= 0.73728$

$= 1 - P(X \le 3)$

$= 1 - 0.26272$

$= 0.73728$

GDC

New O.S. prompts you!

CALCULATOR Functions

(1.) Binomial Probability Distribution function (binompdf)

→ use to find single probability

i.e. $x = 1$, $x = 2$, $x = 3$, $x = 4$, $x = 5$, ⋮ } single terms of binomial expansion

i.e. in part a) of last example $P(X=2)$

(1) Press $\boxed{2nd}$ $\boxed{Vars}$ (DISTR) and select A: binompdf(

enter: binompdf(5, 0.80, 2) ≈ 0.0512

n — probability P(H) — x = 2 (hitting target twice)

(2.) Binomial cummulative (add up) distribution function (binomcdf)

→ use to add up terms of binomial expansion

i.e) in part b) of last example $P(X \leq 3)$

(1) Press $\boxed{2nd}$ $\boxed{Vars}$ (DISTR) and select B: binomcdf

enter: binomcdf(5, 0.8, 3) (this will add $P(x=0) + P(x=1) + P(x=2) + P(x=3)$)

n — P(H) — 3 or less

≈ 0.26272