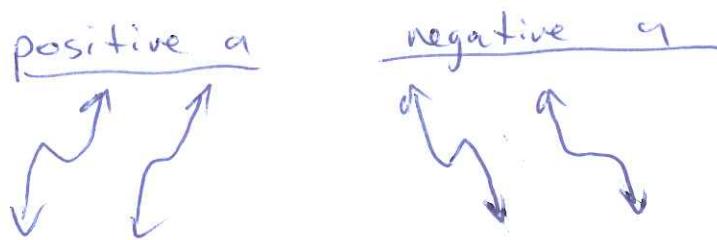


Given the graph of the cubic, write the equations.

- ① Write down the information you are given
- ~~pre~~ x-intercepts
 - y-intercepts
 - any points on the cubic function
 - look @ the "a" value.



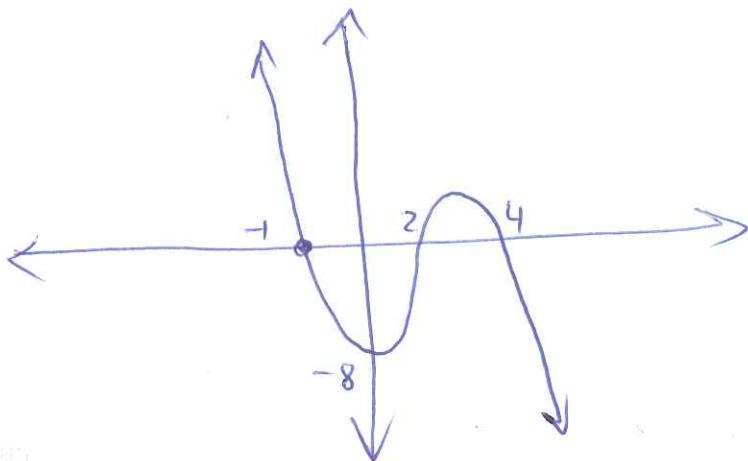
- ② Identify the skeleton equation.

<u>shape</u>	<u>skeleton equation</u>
i) 1 intercept	$f(x) = ax^3$ or $f(x) = a(x-\alpha)^3 + \beta$ α = horizontal shift β = vertical shift
ii) 2 intercepts	$f(x) = a(x-\alpha)^2(x-\beta)$ factors cuts
iii) 3 intercepts	$f(x) = a(x-\alpha)(x-\beta)(x-\gamma)$ where α , β , and γ are the x-intercepts

- ③ Plug in any points to solve for the a-value

Example 1 from class.

Given the graph below, write the general equation for the cubic function.



① Information we know:

x-intercepts

$$x = -1, 2, 4 \quad (y = 0)$$

y-intercepts

$$y = -8 \quad (x = 0)$$

a value should be negative.

② Since there are 3 intercepts, use

$$f(x) = a(x - \alpha)(x - \beta)(x - \gamma) \quad \text{plug in } x\text{-intercepts}$$

$$f(x) = a(x - (-1))(x - 2)(x - 4)$$

$$f(x) = a(x + 1)(x - 2)(x - 4)$$

③ Plug in y-intercept to solve for a $(0, -8)$

$$f(0) = -8 = a(0 + 1)(0 - 2)(0 - 4)$$

$$-8 = a(1)(-2)(-4)$$

$$-8 = a(+8)$$

$$\frac{-8}{8} = \frac{8a}{8}$$

$$-1 = a$$

$\therefore f(x) = -1(x + 1)(x - 2)(x - 4)$ is our equation

next page

Note: This is given in factored form.
To write in general form you must expand.

$$f(x) = -1 \underbrace{(x+1)(x-2)(x-4)}$$

Foil this first

$$= -1 (x^2 - 2x + x - 2)(x-4)$$

$$= -1 (x^2 - x + 2)(x-4)$$

distribute the -1

$$= (-x^2 + x - 2)(x-4)$$

$$= (x-4)(-x^2 + x - 2)$$

(since multiplication
is commutative:
i.e. $2 \cdot 3 = 3 \cdot 2 = 6$)

$$= (x-4)(-x^2 + x - 2) \quad \text{distribute } (x-4) \text{ onto each term.}$$

$$= (x-4)(-x^2) + (x-4)(x) + (x-4)(-2)$$

$$= \overbrace{(x-4)(-x^2)} + \overbrace{(x-4)(x)} + \overbrace{(x-4)(-2)} \quad \text{distribute into brackets}$$

$$= -x^3 + 4x^2 + x^2 - 4x - 2x + 8$$

$$= -x^3 + 5x^2 - 6x + 8 \quad \text{combine like terms}$$

$$f(x) = ax^3 + bx^2 + cx + d$$

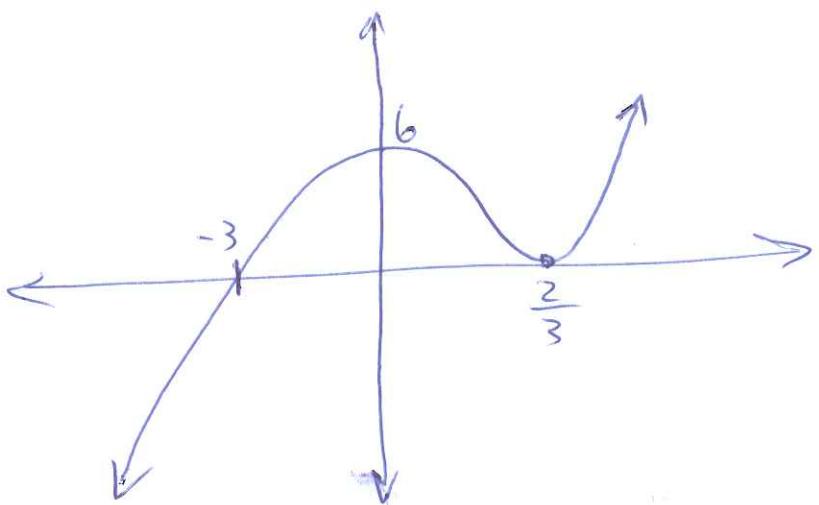
$$\text{where } a = -1$$

$$b = 5$$

$$c = -6$$

$$d = 8$$

Example 2



① x-intercepts:

cuts @ -3 ($y=0$)

touches @ $\frac{2}{3}$

y-intercepts: $(x=0)$

$$y = 6$$

a suggest positive value



② 2 intercepts suggests skeleton equation:

$$f(x) = a(\underbrace{(x-\alpha)^2}_{\text{touch}})(\underbrace{(x-\beta)}_{\text{cut}})$$

↓ plug in information

$$f(x) = a\left(x - \frac{2}{3}\right)^2(x + 3)$$

plugging in x-intercepts

$$f(x) = a\left(x - \frac{2}{3}\right)^2(x + 3)$$

↓ plug in y-intercept
 $(0, 6)$

$$f(0) = 6 = a\left(-\frac{2}{3}\right)^2(0 + 3)$$

$$6 = a\left(\frac{4}{9}\right)(3)$$

$$6 = a\left(\frac{12}{9}\right)$$

$$6 = a\left(\frac{4}{3}\right)$$

$$6 = \frac{4}{3}a$$

$$\frac{6 \cdot 3}{4} = \frac{4a}{4}$$

$$\frac{18}{4} = a$$

$$4.5 = a$$

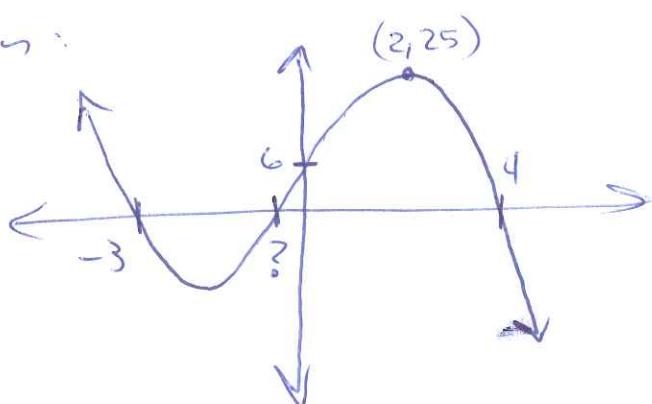
∴ our equation is

$$f(x) = 4.5a\left(x - \frac{2}{3}\right)^2(x + 3)$$

Example 3: If you are not given 1 of the x-intercepts you must find it.

Use 2 equations w/ 2 unknowns to solve.

Given:



① Info we know

x-int ($y=0$)

$x = -3, ?, 4$

y-int ($x=0$)

$y = 6$

a suggests negative shape

② 3 intercepts use the skeleton

$$f(x) = a(x-\alpha)(x-\beta)(x-\gamma)$$

③ Plug in x-intercepts.

(eq1) $f(x) = a(x-(-3))(x-\beta)(x-4)$

Plug in y-intercept
(0, 6)

we don't know
this one.

$$f(0) = 6 = a(0+3)(0-\beta)(0-4)$$
$$6 = a(3)(-\beta)(-4)$$

$$6 = a(-12)(-\beta)$$

(eq2) $6 = a\beta \cdot 12$ (neg x neg = pos)

Solve equation
for 1 variable &
plug into other.

$$\frac{6}{12} = \frac{a\beta \cdot 12}{B \cdot 12}$$

or \rightarrow

$$\frac{6}{12a} = \frac{a\beta \cdot 12}{12a}$$

$$\frac{1}{2\beta} = a$$

$$\frac{1}{2a} = \beta$$

$$f(x) = a(x+3)(x-\beta)(x-4)$$

Substitute

next page

$$f(x) = a(x+3)(x - \frac{1}{2a})(x-4)$$

$$f(2) = 25 = a(2+3)(2 - \frac{1}{2a})(2-4)$$

$$25 = a(5)(2 - \frac{1}{2a})(-2)$$

$$25 = a(-10)(2 - \frac{1}{2a})$$

$$25 = (-10)a(2 - \frac{1}{2a})$$

$$25 = -10(2a - \frac{1}{2a})$$

$$25 = -10(2a - \frac{1}{2})$$

$$25 = -20a + 5$$

$$-5 \quad -5$$

$$\frac{20}{-20} = \frac{-20a}{-20}$$

$$-1 = a$$

$$\therefore \text{Back to eq 2} \quad 6 = a\beta \cdot 12$$

$$6 = -1 \cdot \beta \cdot 12$$

$$6 = -12 \cdot \beta$$

$$\frac{6}{-12} = \beta$$

$$-\frac{1}{2} = \beta$$

$$\therefore \text{Finally, } f(x) = -1(x+3)\left(x - \left(\frac{1}{2}\right)\right)(x-4) \quad \text{or}$$

$$f(x) = -(x+3)\left(x + \frac{1}{2}\right)(x-4)$$

This one was difficult.... but give it a try!