6A Quadratic Equations

Review: $y = f(x) = ax^2 + bx + c$ (expanded form)

"Roots" or "solutions": x-values that make

$$ax^2 + bx + c = 0$$

Methods to solve:

M ① factorize and "null factor law"

If $a \cdot b = 0$ then $a = 0$ or $b = 0$.

Ex: a) $3x^2 + 4x = 0$

$x(3x + 4) = 0$

$\therefore x = 0$ or $3x + 4 = 0$

$3x = -4$

$x = -4/3$

Check:

$3(0)^2 + 4(0) = 0$

$0 = 0$ ✓

$3\left(\frac{-4}{3}\right)^2 + 4\left(\frac{-4}{3}\right) = 0$

$\cancel{3}\left(\frac{16}{\cancel{9}3}\right) - \frac{16}{3} = 0$

$\frac{16}{3} - \frac{16}{3} = 0$

$0 = 0$ ✓

b) $x^2 = 5x - 6$ to 1 side

$\therefore x^2 - 5x + 6 = 0$ foil backwards

$(x - 2)(x - 3) = 0$

$\therefore x - 2 = 0$ or $x - 3 = 0$

$\therefore x = 2$ or 3

c) $x\left(2x - \frac{1}{x}\right) = (-1)x$ HWTOT w/ x

$2x^2 - 1 = -x$

$2x^2 + x - 1 = 0$

$(2x - 1)(x + 1) = 0$

$\therefore 2x - 1 = 0$ or $x + 1 = 0$

$2x = 1$

$x = \frac{1}{2}$ or $x = -1$

$\boxed{2^{nd}}$ Method to solve:

Complete the square:

Ex: If $x^2 = a$ then $x = \pm\sqrt{a}$

a) $\sqrt{(x+2)^2} = \sqrt{7}$

$\therefore$ $x+2 = \pm\sqrt{7}$

$x = -2+\sqrt{7}$ or $-2-\sqrt{7}$ $\quad (-2 \pm \sqrt{7})$

b) $\sqrt{(x+2)^2} = \sqrt{-5}$

No solutions!

Notice in a)

$(x+2)^2 = 7$

$x^2 + 4x + 4 = 7$

$x^2 + 4x + 4 - 7 = 0$

$x^2 + 4x - 3 = 0$

c) $x^2 + 4x - 3 = 0$

$(x \quad 3)(x \quad 1)$; stuck!

$\therefore$ How to go backwards?

1. put x terms on 1 side + constant on other
2. add $\left(\frac{b}{2}\right)^2$ to both sides $(ax^2 + bx + c)$
3. factorize to "complete" square
4. solve

$\Longrightarrow$

$\therefore$ given $x^2 + 4x - 3 = 0$

$x^2 + 4x = 3$ add $\left(\frac{4}{2}\right)^2 = 2^2 = 4$

$x^2 + 4x + 4 = 3 + 4$ "complete square"

$(x+2)(x+2) = 7$

$(x+2)^2 = 7$!

<u>Note:</u> $2x^2 + 10x + 3 = 0$ $\div$ by a first!

$\therefore x^2 + \frac{10x}{2} + \frac{3}{2} = 0$

$x^2 + 5x + \frac{3}{2} = 0$

$x^2 + 5x = -\frac{3}{2}$ add $\left(\frac{5}{2}\right)^2 = \frac{25}{4}$

$x^2 + 5x + \boxed{\frac{5^2}{2^2}} = -\frac{3}{2} + \frac{25}{4}$ factorize

$\left(x + \frac{5}{2}\right)\left(x + \frac{5}{2}\right) = \frac{-3}{4} + \frac{25}{4}$

$\sqrt{\left(x + \frac{5}{2}\right)^2} = \sqrt{\frac{22}{4}}$

$x + \frac{5}{2} = \pm\sqrt{\frac{22}{4}}$

$\therefore x = -\frac{5}{2} \pm \sqrt{\frac{22}{4}}$

<u>You try:</u> $x^2 + 6x + 7 = 0$

$x^2 + 6x = -7$

$x^2 + 6x + \left(\frac{6}{2}\right)^2 = -7 + \left(\frac{6}{2}\right)^2$

$x^2 + 6x + 9 = -7 + 9$

$(x+3)^2 = 2$

$x + 3 = \pm\sqrt{2}$

$x = -3 \pm \sqrt{2}$

Method 3 Quadratic formula.

for $0 = ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(proof on pg 160!)

Previous example: $x^2 + 6x + 7 = 0$

$a = 1$
$b = 6$
$c = 7$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 - 28}}{2}$$

$$x = \frac{-6 \pm \sqrt{8}}{2}$$

$$x = \frac{-6}{2} \pm \frac{\sqrt{4 \cdot 2}}{2}$$

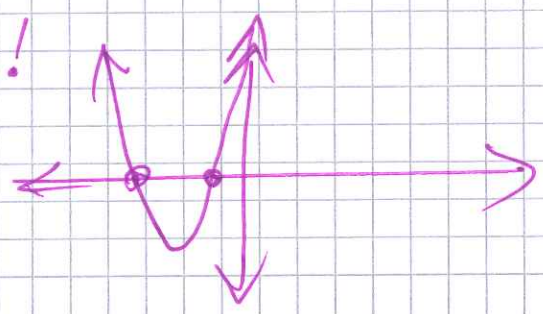
$$x = -3 \pm \frac{\sqrt{4} \cdot \sqrt{2}}{2}$$

$$x = -3 \pm \frac{\cancel{2} \cdot \sqrt{2}}{\cancel{2}}$$

$$x = -3 \pm \sqrt{2} \quad !$$

$$x \approx -1.5857\ldots \text{ or } -4.414\ldots$$

Method 4 GDC!



SMART VIEW

(See website?!)

6B The discriminant

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

discriminant $= b^2 - 4ac = \Delta$

$$\therefore \quad x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$\Delta > 0$

$= \underline{\underline{2 \text{ roots}}}$

$\Delta = 0$

$= 1$ root

$\Delta < 0$

$= \sqrt{-\#}$

$=$ NO REAL solutions

imaginary $i = \sqrt{-1}$

to come :)

Ex: $2x^2 - 7x - 5 = 0$

$b^2 - 4ac -$

$= (-7)^2 - 4(2)(-5)$

$= 49 + 8(+5)$

$= 49 + +35$

$=$ positive

$\therefore$ 2 solutions!

Sign Diagram

6C Graphing Quadratics

See text pg 164 line of symm etc.

See text pg 166

$$y = a(x-h)^2 + k$$

squishy factor — horizontal shift — vertical shift!

$\therefore$ translation vector $= \begin{pmatrix} h \\ k \end{pmatrix}$ and vertex $= V(h, k)$

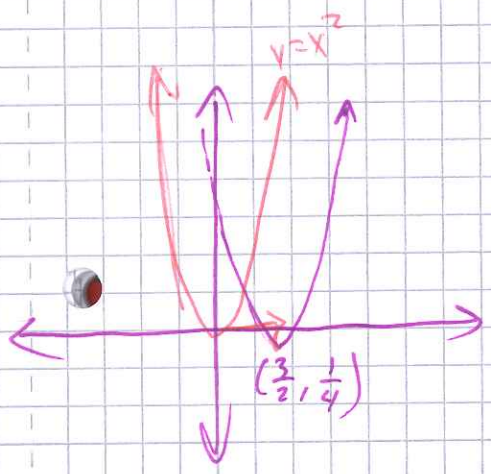
Ex: $y = x^2 - 3x + 2 = 0$

$$x^2 - 3x + \left(\frac{-3}{2}\right)^2 = -2 + \left(\frac{-3}{2}\right)^2$$

$$(x - 3/2)(x - 3/2) = -2 + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = -\frac{8}{4} + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{1}{4}$$

$$\left(x - \frac{3}{2}\right)^2 - \frac{1}{4} = 0 = y$$

$$\begin{pmatrix} +\frac{3}{2} \\ -1/4 \end{pmatrix}$$