Name: KEY

IB Mathematics HL Year 1

Test Ch1

Calculator Section

35 Minutes

34 Points

Please show your full workings and asnwers in the space provided.

1. Three Mathematics books, five English books, four Science books and a dictionary are to be placed on a student's shelf so that the books of each subject remain together.

(a) In how many different ways can the books be arranged?

(4)

(b) In how many of these will the dictionary be next to the Mathematics books?

(3)

(Total 7 marks)

a) 3 math | 5 english | 4 science | 1 dict.

$\underbrace{3! \times 5! \times 4! \times 1!}$

AND 4! ways of ordering the groups of books.

$\therefore 4!(3! \times 5! \times 4! \times 1!) = 414,720$ ways.

b) 3! ways of ordering math text books. Each math book has two ways to place a dictionary next to it.

$\therefore$ (Ways of placing dictionary next to Math text) AND (ways of ordering the other text books)

$= (3! \times 2) \times (3! \times 4! \times 5!) = 207,360$

(half!)

2. When $\left(1+\frac{x}{2}\right)^n$, $n \in \mathbb{N}$, is expanded in ascending powers of x, the coefficient of x^3 is 70.

(a) Find the value of n.

(5)

(b) Hence, find the coefficient of x^2.

(1)

(Total 6 marks)

a) $1^n + \binom{n}{1}1^{n-1}\left(\frac{x}{2}\right)^1 + \binom{n}{2}1^{n-2}\left(\frac{x}{2}\right)^2 + \boxed{\binom{n}{3}1^{n-3}\left(\frac{x}{2}\right)^3} + \ldots$

Note: $1^{n-3} = 1$

$\therefore \binom{n}{3}\left(\frac{x}{2}\right)^3 = \binom{n}{3}\left(\frac{x^3}{2^3}\right)$

$= \binom{n}{3}\left(\frac{1}{8}\right)x^3$

$\therefore$ coefficient is

$\binom{n}{3}\left(\frac{1}{8}\right) = 70$

$\frac{n!}{(n-3)! \cdot 3!}\left(\frac{1}{8}\right) = 70$

$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!} \cdot 6} = 560$

$n(n-1)(n-2) = 3,360$

$\Longrightarrow$ trial and error

$16 \cdot 15 \cdot 14 = 3,360$ or

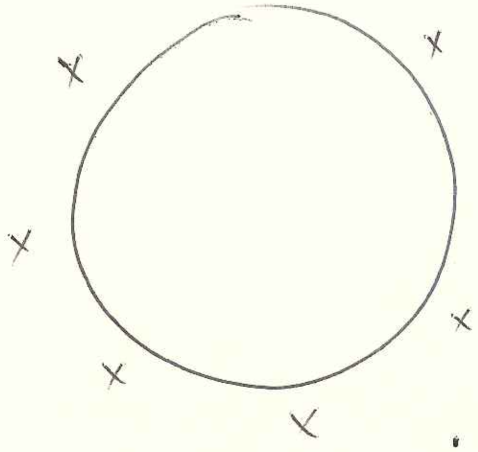
Use GDC:

$\underbrace{x(x-1)(x-2)}_{Y_1} = \underbrace{3,360}_{Y_2}$

[2nd] [trace] [intersect]

3. Six people are to sit at a circular table. Two of the people are not to sit immediately beside each other. Find the number of ways that the six people can be seated.

(Total 5 marks)



Recall in a circle: $(n-1)!$ ways to sit

$\therefore (6-1)! = 5! = 120$ ways to be arranged.

Person 1 and Person 2 (P_1 & P_2) can sit next to eachother in two ways and the remaing 4 in $4!$ ways.

$\therefore \; 2\times 4! = 48$

$\therefore$ Total − # of ways P_1 & P_2 can sit by eachother

$= 120 - 48 = 72$ ways.

4. In the arithmetic series the n[th] term u_n, it is given that $u_4 = 7$ and $u_9 = 22$. Find the minimum value of n so that $u_1 + u_2 + u_3 + \ldots + u_n$ exceeds 10,000.

(5)

$u_4 = u_1 + (n-1)d \qquad u_9 = u_1 + (n-1)d$

$7 = u_1 + (4-1)d \qquad 22 = u_1 + (9-1)d$

(1) $\boxed{7 = u_1 + 3d}$ (2) $\boxed{22 = u_1 + 8d}$

$$\begin{aligned} &\;\;22 = \cancel{u_1} + 8d \\ -&\;\;\;7 = \cancel{u_1} + 3d \\ \hline &\;\;15 = 5d \end{aligned}$$

$\therefore d = 3$

$\therefore 7 = u_1 + 3(3)$

$-2 = u_1$

Now $u_1 + u_2 + u_3 + \ldots + u_n = S_n$

To find when $S_n > 10{,}000$ use

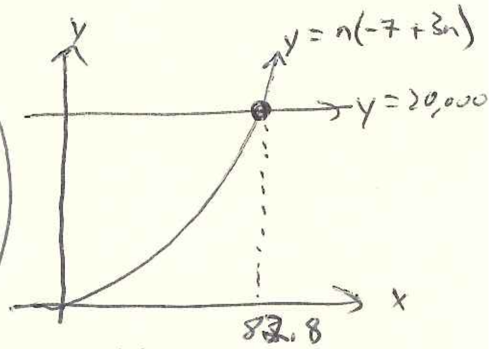
$\frac{n}{2}(2u_1 + (n-1)d) > 10{,}000$

$\frac{n}{2}(2(-2) + (n-1)3) > 10{,}000$

$\frac{n}{2}(-4 + 3n - 3) > 10{,}000$

$\underbrace{n(-7+3n)}_{Y_1} > \underbrace{20{,}000}_{Y_2}$

2nd trace intersect.



$\therefore n \approx 82.8$

$\therefore n = \underline{\underline{83}}$ to exceed 10,000.

PLEASE SHOW WORKING FOR THIS PROBLEM ON SEPARATE PAPER!

5. The diagrams below show the first four squares in a sequence of squares which are subdivided in half. The area of the shaded square A is $\frac{1}{4}$.

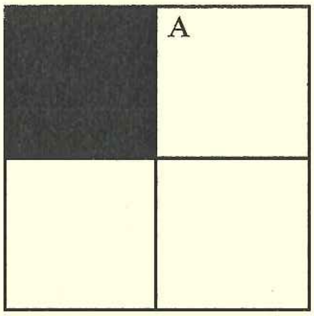


Diagram 1

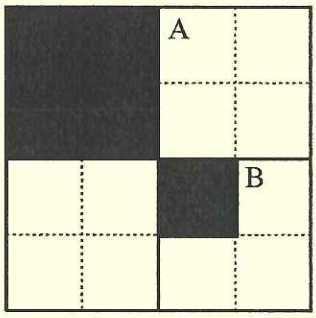


Diagram 2

(see next pg)

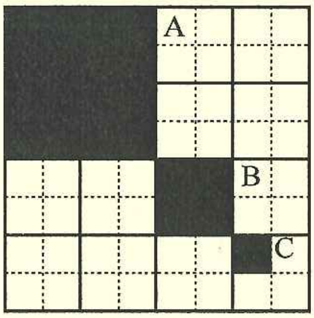


Diagram 3

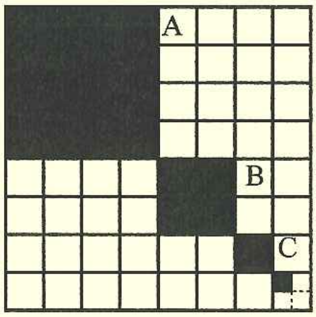


Diagram 4

(a) (i) Find the area of square B and of square C.

(ii) Show that the areas of squares A, B and C are in geometric progression.

(iii) Write down the common ratio of the progression.

(5)

(b) (i) Find the **total** area shaded in diagram 2.

(ii) Find the **total** area shaded in the 8^{th} diagram of this sequence.
Give your answer correct to six significant figures.

(4)

(c) The dividing and shading process illustrated is continued indefinitely.
Find the total area shaded.

(2)
(Total 11 marks)

WORKING FOR #5

a) i) $A_{\text{Square } B} = \frac{1}{4}(A)$ $\qquad A_{\text{square } C} = \frac{1}{4}(B)$

$= \frac{1}{4} \cdot \frac{1}{4} \qquad\qquad = \frac{1}{4} \cdot \frac{1}{16}$

$= \frac{1}{16} \qquad\qquad = \frac{1}{64}$

ii) "Show" the common ratio is equal.

If $A, B, C = \frac{1}{4}, \frac{1}{16}, \frac{1}{64}$ then

$$\frac{\left(\frac{1}{64}\right)}{\left(\frac{1}{16}\right)} = \frac{\left(\frac{1}{16}\right)}{\left(\frac{1}{4}\right)} \quad \cancel{\Rightarrow \left(\frac{1}{4}\right)\left(\frac{1}{64}\right) = \left(\frac{1}{16}\right)\left(\frac{1}{16}\right)}$$

$$\Rightarrow \frac{1}{64} \cdot \frac{16}{1} = \frac{1}{16} \cdot \frac{4}{1}$$

$$\Rightarrow \frac{16}{64} = \frac{4}{16}$$

$$\Rightarrow \frac{1}{4} = \frac{1}{4} \quad \checkmark$$

iii) $r = \frac{1}{4}$

b) i) $\frac{1}{4} + \frac{1}{16} = \frac{4}{16} + \frac{1}{16} = \frac{5}{8}$

ii) Use $S_8 = \frac{u_1(1 - r^n)}{1 - r}$

$$= \frac{\left(\frac{1}{4}\right)\left(1 - \left(\frac{1}{4}\right)^8\right)}{1 - \frac{1}{4}}$$

$\Rightarrow$ GDC $\approx 0.33332824 7 \ldots$

≈ 0.333328 (6 S.F)

c) $S_\infty = \frac{u_1}{1 - r}$

$$= \frac{(1/4)}{1 - \left(\frac{1}{4}\right)}$$

$$= \frac{0.25}{0.75} \quad = \frac{1}{3}$$

$= 0.\bar{3}$.