

- 7 If X is the number of questions Raj answers correctly, then X is binomial. There are $n = 10$ independent trials with probability $p = \frac{1}{5}$ of a correct answer for each.

$$\begin{aligned} P(\text{Raj passes}) &= P(X \geq 7) \\ &= 1 - P(X \leq 6) \\ &\approx 1 - 0.999136 \\ &\approx 0.000864 \quad \{\text{or about 9 in 10 000}\} \end{aligned}$$

- 8 $P(M \text{ wins a game against J}) = \frac{2}{3} \therefore P(M \text{ wins}) = \frac{2}{3} \quad P(J \text{ wins}) = \frac{1}{3}$
 $P(J \text{ wins a set 6 games to 4}) = P(J \text{ wins 5 of the first 9 games and J wins the 10th game})$
 this is binomial with $n = 9$ trials of probability $p = \frac{1}{3}$
 $\approx 0.1024 \times \frac{1}{3}$
 ≈ 0.0341

- 9 If there are n dice thrown, $P(\text{no sixes}) = \left(\frac{5}{6}\right)^n$
 $\therefore P(\text{at least 1 six}) = 1 - \left(\frac{5}{6}\right)^n$

$$\begin{aligned} \therefore \text{need to find the smallest integer } n \text{ such that } 1 - \left(\frac{5}{6}\right)^n \geq 0.5 \\ \therefore \left(\frac{5}{6}\right)^n \leq 0.5 \\ \therefore n \log\left(\frac{5}{6}\right) \leq \log(0.5) \\ \therefore n \geq \frac{\log(0.5)}{\log\left(\frac{5}{6}\right)} \quad \{\log\left(\frac{5}{6}\right) < 0\} \\ \therefore n \geq 3.80 \end{aligned}$$

\therefore at least 4 dice are needed.

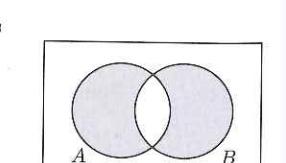
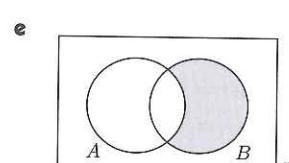
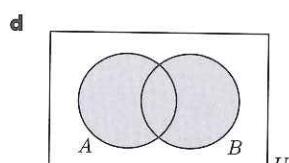
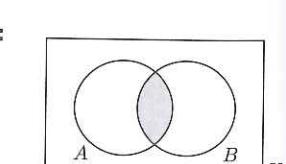
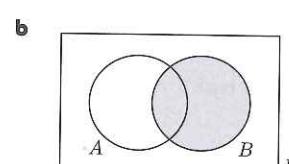
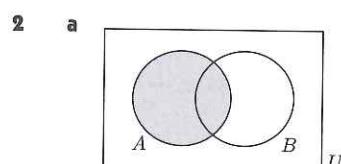
EXERCISE 15I.1

1 a $A = \{1, 2, 3, 6\}$, $B = \{2, 4, 6, 8, 10\}$

b i $n(A) = 4$

ii $A \cup B = \{1, 2, 3, 4, 6, 8, 10\}$

iii $A \cap B = \{2, 6\}$



3 a Total number in the class = $3 + 5 + 17 + 4 = 29$

b Number who study both = 17 {the intersection}

c Number who study at least one = $5 + 17 + 4 = 26$ {the union}

d Number who study only Chemistry = 5

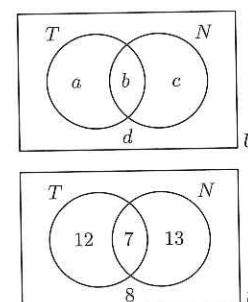
4 a Total number in the survey = $37 + 9 + 15 + 4 = 65$

b Number who liked both = 9 {the intersection}

c Number who liked neither = 4

d Number who liked exactly one = $37 + 15 = 52$

5



a $P(\text{plays tennis})$

$$= \frac{12 + 7}{40} = \frac{19}{40}$$

d $P(\text{plays one and only one})$

$$= \frac{12 + 13}{40} = \frac{25}{40} = \frac{5}{8}$$

T represents those playing tennis
 N represents those playing netball

$$\therefore \begin{cases} a + b + c + d = 40 \\ a + b = 19 \\ b + c = 20 \\ d = 8 \end{cases}$$

$$\begin{aligned} \text{So, } a + b + c &= 32 \\ \therefore 19 + c &= 32 \text{ and } a + 20 = 32 \\ \therefore c &= 13 \text{ and } a = 12 \end{aligned}$$

Hence, $12 + b = 19$

$$\therefore b = 7$$

b $P(\text{does not play netball})$

$$= \frac{12 + 8}{40} = \frac{1}{2}$$

c $P(\text{plays at least one})$

$$= \frac{12 + 7 + 13}{40} = \frac{32}{40} = \frac{4}{5}$$

e $P(\text{plays netball, but not tennis}) = \frac{13}{40}$

f $P(\text{plays tennis given plays netball})$
 $= \frac{7}{7 + 13} = \frac{7}{20}$

C represents men who gave chocolates.
 F represents men who gave flowers.

$$\therefore \begin{cases} a + b + c + d = 50 \\ a + b = 31 \\ b + c = 12 \\ b = 5 \end{cases}$$

Thus $c = 7$, $a = 26$ and $26 + 5 + 7 + d = 50 \therefore d = 12$

a $P(C \text{ or } F)$

$$= \frac{26 + 5 + 7}{50} = \frac{38}{50} \text{ or } \frac{19}{25}$$

b $P(C \text{ but not } F)$

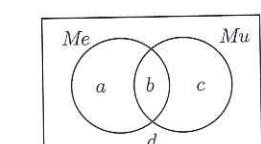
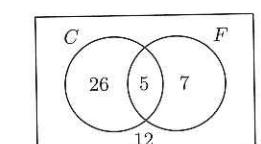
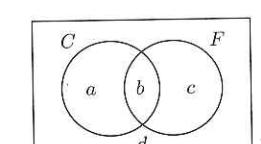
$$= \frac{26}{50} = \frac{13}{25}$$

c $P(\text{neither } C \text{ nor } F)$

$$= \frac{12}{50} = \frac{6}{25}$$

d $P(F \text{ given that } C')$

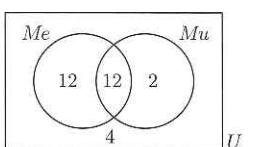
$$= \frac{7}{7 + 12} = \frac{7}{19}$$



Me represents children who had measles.

Mu represents children who had mumps.

$$\therefore \begin{cases} a + b + c + d = 30 \\ a + b = 24 \\ b = 12 \\ a + b + c = 26 \end{cases}$$



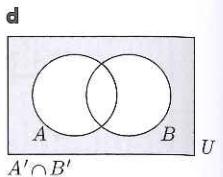
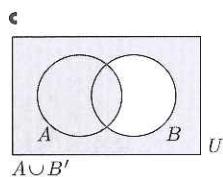
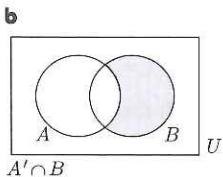
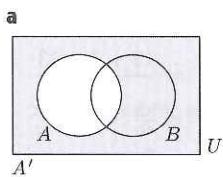
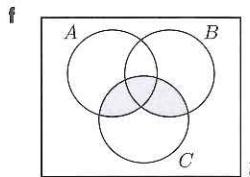
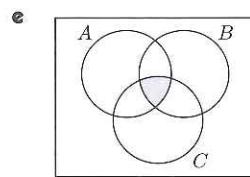
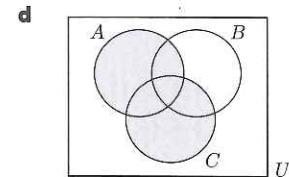
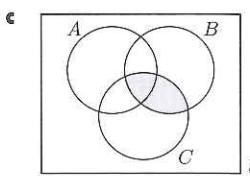
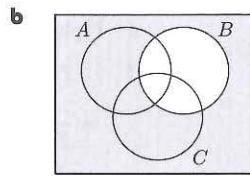
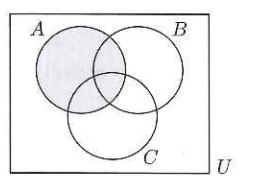
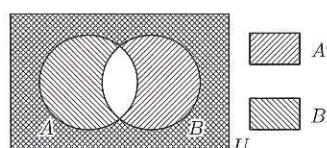
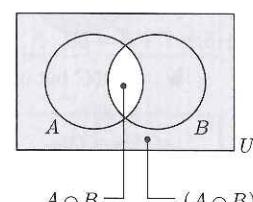
$$\begin{aligned} \therefore 26 + d &= 30 & \therefore d &= 4 \\ 24 + c &= 26 & \therefore c &= 2 \\ \text{and } a + 12 &= 24 & \therefore a &= 12 \end{aligned}$$

a $P(Mu)$

$$\begin{aligned} &= \frac{14}{30} \\ &= \frac{7}{15} \\ &\text{c} \quad P(\text{neither } Mu \text{ nor } Me) \\ &= \frac{4}{30} \\ &= \frac{2}{15} \end{aligned}$$

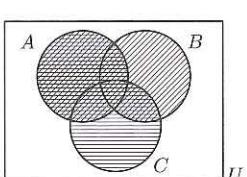
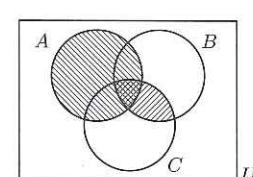
b $P(Mu, \text{ but not } Me)$

$$\begin{aligned} &= \frac{2}{30} \\ &= \frac{1}{15} \\ &\text{d} \quad P(Me \text{ given } Mu) \\ &= \frac{12}{14} \\ &= \frac{6}{7} \end{aligned}$$

8**9****EXERCISE 15I.2****1**

So $A' \cup B'$ is the region containing either type of shading.

Thus, as the regions are the same, $(A \cap B)' = A' \cup B'$ is verified.

b

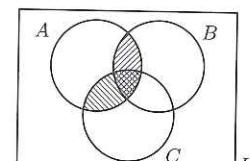
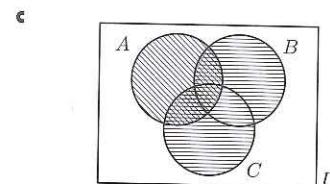
$A \cup (B \cap C)$ consists of the shaded region



$(A \cup B) \cap (A \cup C)$ consists of the 'double shaded' region.

As the two regions are identical

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ is verified.



$A \cap (B \cup C)$ consists of the double shaded region



As the regions are identical, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ is verified.

$(A \cap B) \cup (A \cap C)$ consists of the region shaded. (all forms and)

2 **a** $A = \{7, 14, 21, 28, 35, \dots, 98\}$

$$B = \{5, 10, 15, 20, 25, \dots, 95\}$$

$$\text{i} \quad \text{as } 98 = 7 \times 14, n(A) = 14$$

$$\text{iii} \quad A \cap B = \{35, 70\} \therefore n(A \cap B) = 2$$

$$\text{iv} \quad A \cup B = \{5, 7, 10, 14, 15, 20, 21, 25, 28, 30, 35, 40, 42, 45, 49, 50, 55, 56, 60, 63, 65, 70, 75, 77, 80, 84, 85, 90, 91, 95, 98\} \therefore n(A \cup B) = 31$$

b $n(A) + n(B) - n(A \cap B)$

$$\begin{aligned} &= 14 + 19 - 2 \\ &= 31 \\ &= n(A \cup B) \quad \checkmark \end{aligned}$$

c From the diagram, $n(A) + n(B) - n(A \cap B)$

$$\begin{aligned} &= (a + b) + (b + c) - b \\ &= a + b + c \\ &= n(A \cup B) \end{aligned}$$

3 **a** **i** $P(B)$

$$\begin{aligned} &= \frac{n(B)}{n(U)} \\ &= \frac{b+c}{a+b+c+d} \end{aligned}$$

ii $P(A \text{ and } B)$

$$\begin{aligned} &= \frac{n(A \cap B)}{n(U)} \\ &= \frac{b}{a+b+c+d} \end{aligned}$$

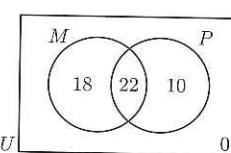
iii $P(A \text{ or } B)$

$$\begin{aligned} &= \frac{n(A \cup B)}{n(U)} \\ &= \frac{a+b+c}{a+b+c+d} \end{aligned}$$

iv $P(A) + P(B) - P(A \text{ and } B) = \frac{a+b+b+c-b}{a+b+c+d}$

$$= \frac{a+b+c}{a+b+c+d}$$

b $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ {using **iii** and **iv**}

EXERCISE 15J**1**

So 22 study both.

b **i** $P(M \text{ but not } P)$

$$\begin{aligned} &= \frac{18}{50} \\ &= \frac{9}{25} \\ &= \frac{22}{40} \\ &= \frac{11}{20} \end{aligned}$$

ii $P(P \text{ given } M)$

$$\begin{aligned} &= \frac{22}{18+22} \\ &= \frac{22}{40} \\ &= \frac{11}{20} \end{aligned}$$

$$a + b + c + d = 40 \quad \dots (1)$$

$$a + b = 23 \quad \dots (2)$$

$$b + c = 18 \quad \dots (3)$$

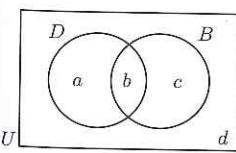
$$a + b + c = 26 \quad \dots (4)$$

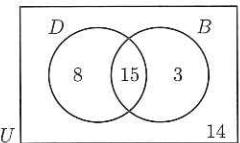
$\therefore d = 14$ {using (1) and (4)}

$23 + c = 26$ and $a + 18 = 26$

$\therefore c = 3$ and $a = 8$

Thus $b = 18 - c = 15$

2

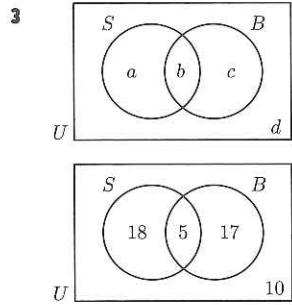


a $P(D \text{ and } B)$
 $= \frac{15}{40}$
 $= \frac{3}{8}$

c $P(D, \text{ but not } B)$
 $= \frac{8}{40}$
 $= \frac{1}{5}$

b $P(\text{neither } D \text{ nor } B)$
 $= \frac{14}{40}$
 $= \frac{7}{20}$

d $P(B \text{ given } D)$
 $= \frac{15}{23}$



$$a + b + c + d = 50$$

$$a + b = 23$$

$$b + c = 22$$

$$b = 5$$

$$\therefore c = 17, a = 18$$

$$\text{and } 18 + 5 + 17 + d = 50$$

$$\therefore d = 10$$

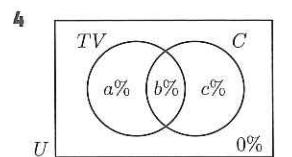
a $P(\text{not } B)$
 $= P(B')$
 $= \frac{28}{50}$
 $= \frac{14}{25}$

b $P(B \text{ or } S)$
 $= \frac{18 + 5 + 17}{50}$
 $= \frac{40}{50}$
 $= \frac{4}{5}$

c $P(\text{neither } B \text{ nor } S)$
 $= \frac{10}{50}$
 $= \frac{1}{5}$

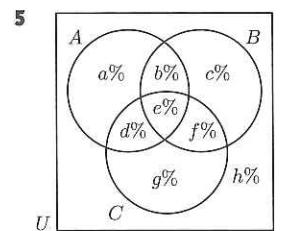
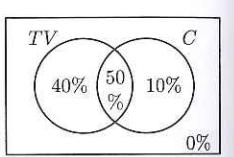
d $P(B, \text{ given } S)$
 $= \frac{5}{18 + 5}$
 $= \frac{5}{23}$

e $P(S, \text{ given } B')$
 $= \frac{18}{18 + 10}$
 $= \frac{18}{28}$
 $= \frac{9}{14}$



$$\left\{ \begin{array}{l} a + b + c = 100 \\ a + b = 90 \quad \therefore c = 10 \text{ and } a = 40 \\ b + c = 60 \quad \therefore b = 50 \end{array} \right.$$

$$P(\text{TV, given } C) = \frac{50}{50 + 10} = \frac{5}{6}$$



$$a + b + c + d + e + f + g + h = 100$$

$$a + b + d + e = 20$$

$$b + c + e + f = 16$$

$$d + e + f + g = 14$$

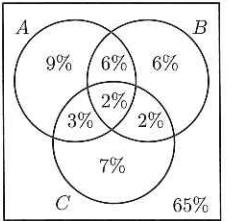
$$b + e = 8$$

$$d + e = 5$$

$$e + f = 4$$

$$e = 2$$

$$\therefore e = 2, f = 2, d = 3, b = 6, \left\{ \begin{array}{l} a + 6 + 3 + 2 = 20 \\ 6 + c + 2 + 2 = 16 \\ 3 + 2 + 2 + g = 14 \end{array} \right. \therefore \left\{ \begin{array}{l} a = 9 \\ c = 6 \\ g = 7 \end{array} \right.$$



a $P(\text{none})$
 $= \frac{65}{100}$
 $= \frac{13}{20}$

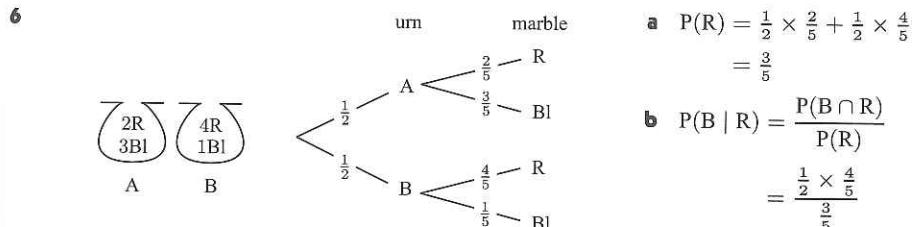
b $P(\text{at least one})$
 $= 1 - P(\text{none})$
 $= 1 - \frac{13}{20}$
 $= \frac{7}{20}$

c $P(\text{exactly one})$
 $= \frac{9 + 6 + 7}{100}$
 $= \frac{22}{100}$
 $= \frac{11}{50}$

d $P(A \text{ or } B)$
 $= \frac{9+6+6+3+2+2}{100}$
 $= \frac{28}{100}$
 $= \frac{7}{25}$

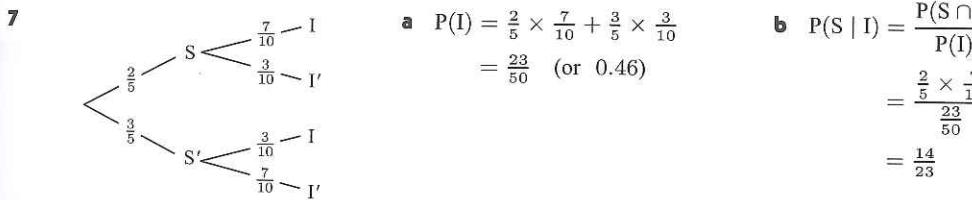
e $P(A, \text{ given at least one})$
 $= \frac{9+6+2+3}{35}$
 $= \frac{20}{35}$
 $= \frac{4}{7}$

f $P(C, \text{ given } A \text{ or } B \text{ or both})$
 $= \frac{3+2+2}{9+6+6+3+2+2}$
 $= \frac{7}{28}$
 $= \frac{1}{4}$



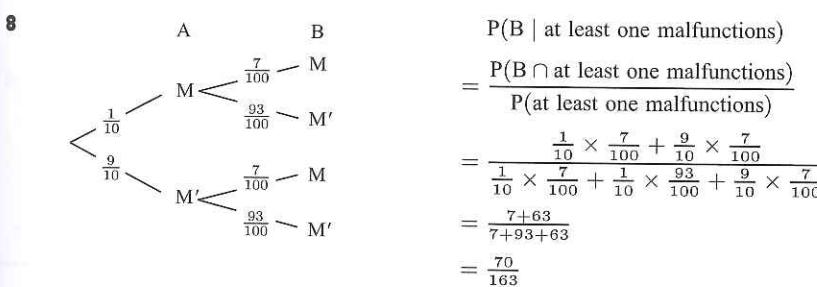
a $P(R) = \frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{4}{5}$
 $= \frac{3}{5}$

b $P(B | R) = \frac{P(B \cap R)}{P(R)}$
 $= \frac{\frac{1}{2} \times \frac{4}{5}}{\frac{3}{5}}$
 $= \frac{2}{3}$



a $P(I) = \frac{2}{5} \times \frac{7}{10} + \frac{3}{5} \times \frac{3}{10}$
 $= \frac{23}{50}$ (or 0.46)

b $P(S | I) = \frac{P(S \cap I)}{P(I)}$
 $= \frac{\frac{2}{5} \times \frac{7}{10}}{\frac{23}{50}}$
 $= \frac{14}{23}$



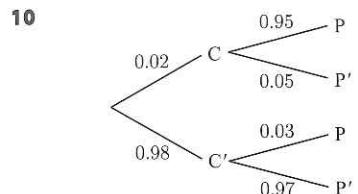
$$\begin{aligned} & P(B | \text{at least one malfunctions}) \\ &= \frac{P(B \cap \text{at least one malfunctions})}{P(\text{at least one malfunctions})} \\ &= \frac{\frac{1}{10} \times \frac{7}{100} + \frac{9}{10} \times \frac{7}{100}}{\frac{1}{10} \times \frac{7}{100} + \frac{1}{10} \times \frac{93}{100} + \frac{9}{10} \times \frac{7}{100}} \\ &= \frac{7+63}{7+93+63} \\ &= \frac{70}{163} \end{aligned}$$

9 $P(B) = 0.5, P(G) = 0.6, P(G | B) = 0.9$, where B is “the boy eats his lunch” and G is “the girl eats her lunch”

a $P(\text{both eat lunch})$
 $= P(B \cap G)$
 $= P(G | B) \times P(B)$
 $= 0.9 \times 0.5$
 $= 0.45$

b $P(B | G)$
 $= \frac{P(B \cap G)}{P(G)}$
 $= \frac{0.45}{0.6}$
 $= 0.75$

c $P(\text{at least one eats lunch})$
 $= P(B \cup G)$
 $= P(B) + P(G) - P(B \cap G)$
 $= 0.5 + 0.6 - 0.45$
 $= 0.65$



11 The coins are H, H T, T and H, T.

$$\text{Any one of these 6 faces could be seen uppermost, } \therefore P(\text{falls H}) = \frac{3}{6} = \frac{1}{2}$$

$$\begin{aligned} \text{Now } P(\text{HH coin} \mid \text{falls H}) &= \frac{P(\text{HH coin} \cap \text{falls H})}{P(\text{falls H})} \\ &= \frac{P(\text{HH})}{P(\text{falls H})} \\ &= \frac{\frac{1}{3}}{\frac{1}{2}} \\ &= \frac{2}{3} \end{aligned}$$

EXERCISE 15K

1 $P(R \cap S)$

$$\begin{aligned} &= P(R) + P(S) - P(R \cup S) \\ &= 0.4 + 0.5 - 0.7 \\ &= 0.2 \end{aligned}$$

So, $P(R \cap S) = P(R) \times P(S)$ and hence R and S are independent events.

2 a $P(A \cap B)$

$$\begin{aligned} &= P(A) + P(B) - P(A \cup B) \\ &= \frac{2}{5} + \frac{1}{3} - \frac{1}{2} \\ &= \frac{7}{30} \end{aligned}$$

b $P(B \mid A)$

$$\begin{aligned} &= \frac{P(B \cap A)}{P(A)} \\ &= \frac{\frac{7}{30}}{\frac{2}{5}} \\ &= \frac{7}{12} \end{aligned}$$

c $P(A \mid B)$

$$\begin{aligned} &= \frac{P(A \cap B)}{P(B)} \\ &= \frac{\frac{7}{30}}{\frac{1}{3}} \\ &= \frac{7}{10} \end{aligned}$$

A and B are not independent as $P(A \mid B) \neq P(A)$.

3 a As X and Y are independent

$$\begin{aligned} P(X \cap Y) &= P(X) \times P(Y) \\ &= 0.5 \times 0.7 \\ &= 0.35 \end{aligned}$$

$\therefore P(\text{both } X \text{ and } Y) = 0.35$

c $P(\text{neither } X \text{ nor } Y)$

$$= 0.15$$

e $P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{0.35}{0.70} = \frac{1}{2}$

4 P(at least one solves it)

$$= 1 - P(\text{no-one solves it})$$

$$= 1 - P(A' \text{ and } B' \text{ and } C')$$

$$= 1 - \frac{2}{5} \times \frac{1}{3} \times \frac{1}{2}$$

$$= 1 - \frac{1}{15}$$

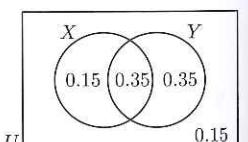
$$= \frac{14}{15}$$

b $P(X \text{ or } Y)$

$$\begin{aligned} &= P(X \cup Y) \\ &= P(X) + P(Y) - P(X \cap Y) \\ &= 0.5 + 0.7 - 0.35 \\ &= 0.85 \end{aligned}$$

d $P(X \text{ but not } Y)$

$$= 0.15$$



a $P(P)$

$$\begin{aligned} &= 0.02 \times 0.95 + 0.98 \times 0.03 \\ &= 0.0484 \end{aligned}$$

b $P(C \mid P)$

$$\begin{aligned} &= \frac{P(C \cap P)}{P(P)} \\ &= \frac{0.02 \times 0.95}{0.0484} \\ &\approx 0.393 \end{aligned}$$

5 a $P(\text{at least one } 6)$

$$\begin{aligned} &= 1 - P(\text{no 6s}) \\ &= 1 - P(6' \text{ and } 6' \text{ and } 6') \\ &= 1 - \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \\ &= 1 - \frac{125}{216} \\ &= \frac{91}{216} \end{aligned}$$

b $P(\text{at least one 6 in } n \text{ throws})$

$$\begin{aligned} &= 1 - \left(\frac{5}{6}\right)^n \\ \text{So we want } 1 - \left(\frac{5}{6}\right)^n &> 0.99 \\ \therefore -\left(\frac{5}{6}\right)^n &> -0.01 \\ \therefore \left(\frac{5}{6}\right)^n &< 0.01 \\ \therefore n \log\left(\frac{5}{6}\right) &< \log(0.01) \end{aligned}$$

$$\therefore n > \frac{\log(0.01)}{\log\left(\frac{5}{6}\right)} \quad \{ \text{as } \log\left(\frac{5}{6}\right) < 0 \}$$

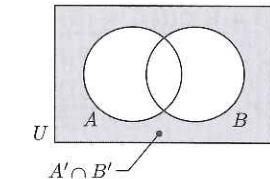
$$\therefore n > 25.2585$$

$$\therefore n = 26$$

6 A and B are independent, so $P(A \cap B) = P(A) P(B)$ (1)

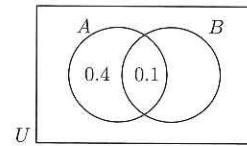
Now $P(A' \cap B')$

$$\begin{aligned} &= 1 - P(A \cup B) \\ &= 1 - [P(A) + P(B) - P(A \cap B)] \\ &= 1 - P(A) - P(B) + P(A \cap B) \\ &= 1 - P(A) - P(B) + P(A) P(B) \quad \{ \text{using (1)} \} \\ &= [1 - P(A)][1 - P(B)] \\ &= P(A') P(B') \end{aligned}$$



$\therefore A'$ and B' are also independent.

7



$\therefore P(A) = 0.5$ and $P(A \cap B) = P(A) \times P(B)$ {A and B are independent}

$$\therefore 0.1 = 0.5 \times P(B)$$

$$\therefore P(B) = 0.2$$

$$\begin{aligned} \text{Now } P(A \cup B') &= P(A) + P(B') - P(A \cap B') \\ &= 0.5 + 0.8 - 0.4 \\ &= 0.9 \end{aligned}$$

8 a i $P(C \mid D) = \frac{P(C \cap D)}{P(D)}$, so $P(C \cap D) = P(C \mid D) P(D)$

Similarly, $P(C \cap D') = P(C \mid D') P(D')$

Now $P(C \cap D) + P(C \cap D') = P(C)$

$\therefore P(C \mid D) P(D) + P(C \mid D') P(D') = P(C)$

$$\therefore \frac{6}{13} P(D) + \frac{3}{7} [1 - P(D)] = \frac{9}{20}$$

$$\therefore \frac{6}{13} P(D) + \frac{3}{7} - \frac{3}{7} P(D) = \frac{9}{20}$$

$$\therefore \frac{3}{91} P(D) = \frac{3}{140}$$

$$\therefore P(D) = \frac{91}{140} \text{ or } \frac{13}{20}$$

ii $P(C \cap D) = P(C \mid D) P(D) = \frac{6}{13} \times \frac{13}{20} = \frac{3}{10}$

Now $P(C' \cup D') = 1 - P(C \cap D)$

$$= 1 - \frac{3}{10} = \frac{7}{10}$$

b $P(C \cap D) = \frac{3}{10}$ and $P(C) P(D) = \frac{9}{20} \times \frac{13}{20} = \frac{117}{400}$

$\therefore C$ and D are not independent as $P(C \cap D) \neq P(C) P(D)$