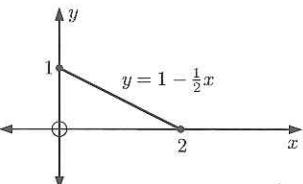


Chapter 24

STATISTICAL DISTRIBUTIONS OF CONTINUOUS RANDOM VARIABLES

EXERCISE 24A

1 a



From the graph, we see that $f(x) \geq 0$ for $0 \leq x \leq 2$.

$$\text{Area} = \frac{1}{2} \times 2 \times 1 = 1 \quad \checkmark$$

$$\text{or } \int_0^2 (1 - \frac{1}{2}x) dx = [x - \frac{1}{4}x^2]_0^2 = (2 - \frac{1}{4}(4)) = 1 \quad \checkmark$$

$\therefore f(x)$ is a probability density function.

$$\text{b } \mu = \int_0^2 x f(x) dx$$

$$= \int_0^2 (x - \frac{1}{2}x^2) dx$$

$$= [\frac{1}{2}x^2 - \frac{1}{6}x^3]_0^2$$

$$= (\frac{1}{2}(4) - \frac{1}{6}(8)) - (0 - 0)$$

$$= \frac{2}{3}$$

$$2 \text{ a } \int_0^4 ax(x-4) dx = 1$$

$$\therefore a \int_0^4 (x^2 - 4x) dx = 1$$

$$\therefore a \left[\frac{x^3}{3} - \frac{4x^2}{2} \right]_0^4 = 1$$

$$\therefore a (\frac{64}{3} - 32) = 1$$

$$\therefore a (-\frac{32}{3}) = 1$$

$$\therefore a = -\frac{3}{32}$$

$$\text{c } P(0 \leq X \leq 1)$$

$$= \int_0^1 -\frac{3}{32}x(x-4) dx$$

$$= \int_0^1 (-\frac{3}{32}x^2 + \frac{3}{8}x) dx$$

$$= [-\frac{1}{32}x^3 + \frac{3}{16}x^2]_0^1$$

$$= -\frac{1}{32} + \frac{3}{16}$$

$$= \frac{5}{32}$$

$$3 \text{ a } \int_0^5 kx^2(x-6) dx = 1$$

$$\therefore k \int_0^5 (x^3 - 6x^2) dx = 1$$

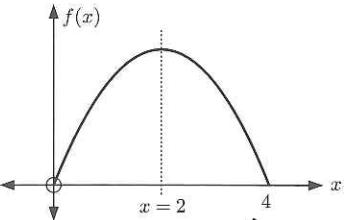
$$\therefore k [\frac{1}{4}x^4 - \frac{6}{3}x^3]_0^5 = 1$$

$$\therefore k (\frac{625}{4} - 250) = 1$$

$$\therefore k (-\frac{375}{4}) = 1$$

$$\therefore k = -\frac{4}{375}$$

$$\text{b } f(x) = -\frac{3}{32}x(x-4), \quad 0 \leq x \leq 4$$



$$\text{d } \mu = \int_0^4 x f(x) dx$$

$$= \int_0^4 -\frac{3}{32}x^2(x-4) dx$$

$$= -\frac{3}{32} \int_0^4 (x^3 - 4x^2) dx$$

$$= -\frac{3}{32} [\frac{1}{4}x^4 - \frac{4}{3}x^3]_0^4$$

$$= -\frac{3}{32} (\frac{1}{4}(4)^4 - \frac{4}{3}(4)^3)$$

$$= -\frac{3}{32} (4^3 - \frac{4}{3} \times 4^3)$$

$$= -\frac{3}{32} (-\frac{64}{3}) = 2$$

$$\text{b } E(X) = \int_0^5 x f(x) dx$$

$$= \int_0^5 -\frac{4}{375}x^3(x-6) dx$$

$$= 3\frac{1}{3} \quad \{\text{using technology}\}$$

$$\text{c } P(3 \leq X \leq 5) = \int_3^5 -\frac{4}{375}x^2(x-6) dx$$

$$= 0.64 \quad \{\text{using technology}\}$$

$$4 \quad P(X \leq \frac{2}{3}) = \frac{1}{243}$$

$$\therefore \int_0^{\frac{2}{3}} ax^4 dx = \frac{1}{243}$$

$$\therefore [\frac{1}{5}ax^5]_0^{\frac{2}{3}} = \frac{1}{243}$$

$$\therefore \frac{1}{5}a \times (\frac{2}{3})^5 = \frac{1}{243}$$

$$\therefore \frac{1}{5}a \times \frac{32}{243} = \frac{1}{243}$$

$$\therefore a = \frac{5}{32}$$

$$\text{So, } f(x) = \frac{5}{32}x^4, \quad 0 \leq x \leq k$$

$$\therefore \int_0^k \frac{5}{32}x^4 dx = 1$$

$$\therefore [\frac{1}{5}x^5]_0^k = 1$$

$$\therefore \frac{1}{5}k^5 = 1$$

$$\therefore k^5 = 32$$

$$\therefore k = 2$$

$$5 \text{ a } \int_0^4 k\sqrt{x} dx = 1$$

$$\therefore k \int_0^4 x^{\frac{1}{2}} dx = 1$$

$$\therefore k [\frac{2}{3}x^{\frac{3}{2}}]_0^4 = 1$$

$$\therefore \frac{2}{3}k (4^{\frac{3}{2}} - 0) = 1$$

$$\therefore \frac{2}{3}k(8) = 1$$

$$\therefore k = \frac{3}{16}$$

$$\text{c } P(1 \leq X \leq 2.5) = \int_1^{2.5} \frac{3}{16}\sqrt{x} dx$$

$$\approx 0.369$$

{using technology}

$$\text{d } P(0 \leq X \leq a) = P(a \leq X \leq 4)$$

Since $P(0 \leq X \leq 4) = 1$,

$$P(0 \leq X \leq a) + P(a \leq X \leq 4) = 1$$

$$\therefore P(0 \leq X \leq a) = \frac{1}{2} = P(a \leq X \leq 4)$$

$$\therefore \int_0^a \frac{3}{16}x^{\frac{1}{2}} dx = \frac{1}{2}$$

$$\therefore \frac{3}{8} [\frac{2}{3}x^{\frac{3}{2}}]_0^a = 1$$

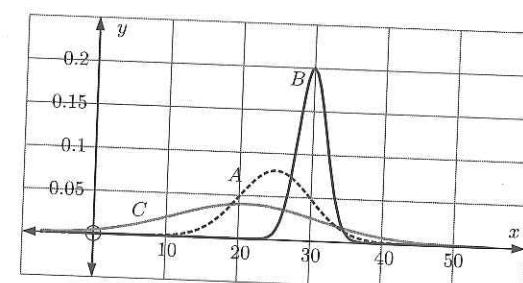
$$\therefore \frac{1}{4} (a^{\frac{3}{2}} - 0^{\frac{3}{2}}) = 1$$

$$\therefore a^{\frac{3}{2}} = 4$$

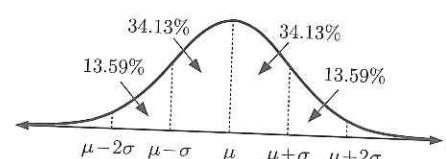
$$\therefore a = \sqrt[3]{16} \approx 2.52 \text{ hours}$$

EXERCISE 24B.1

1

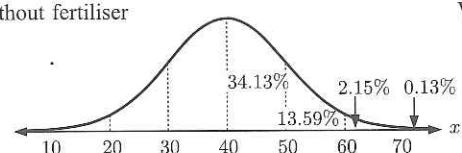


3

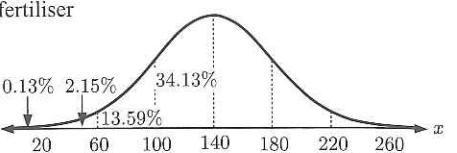


a $P(\text{value between } \mu - \sigma \text{ and } \mu + \sigma) \approx 34.13\% + 34.13\% \approx 0.683$

b $P(\text{value between } \mu \text{ and } \mu + 2\sigma) \approx 34.13\% + 13.59\% \approx 0.477$

4 Without fertiliser


With fertiliser



a $P(\text{without and } X < 50)$

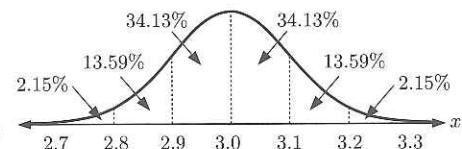
$\approx 50\% + 34.13\%$
 $\approx 84.1\%$

c i $P(\text{with and } 20 \leq X \leq 60)$

$\approx 2.15\%$

d i $P(\text{with and } X \geq 60)$

$\approx 13.59\% + 34.13\% + 50\%$
 $\approx 97.7\%$

5


b $P(\text{with and } X < 60)$

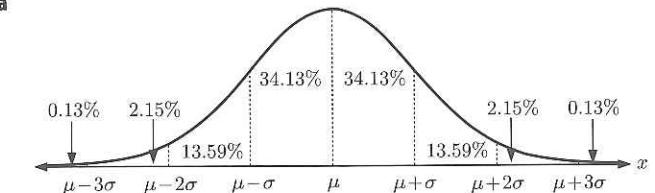
$\approx 0.13\% + 2.15\%$
 $\approx 2.28\%$

ii $P(\text{without and } 20 \leq X \leq 60)$

$\approx 2(34.13\% + 13.59\%)$
 $\approx 95.4\%$

ii $P(\text{without and } X \geq 60)$

$\approx 2.15\% + 0.13\%$
 $\approx 2.28\%$

6 a


84% of the crop weigh more than 152 g $\therefore \mu - \sigma = 152$

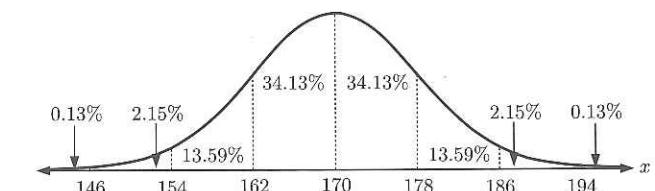
16% of the crop weigh more than 200 g $\therefore \mu + \sigma = 200 \dots (1)$

Adding: $\frac{2\mu}{2} = 352$, and so $\mu = 176$ g

Substituting $\mu = 176$ into (1) gives $\sigma = 200 - \mu = 24$ g.

b For $\mu = 176$ g and $\sigma = 24$ g, 152 g = $\mu - \sigma$, and 224 g = $\mu + 2\sigma$.

\therefore between 152 g and 224 g, the percentage is $34.13\% + 34.13\% + 13.59\% \approx 81.9\%$

7


a i $P(162 < X < 170) \approx 34.1\%$

iii $P(170 < X < 186) \approx 34.13\% + 13.59\%$
 $\approx 47.7\%$

b i $P(178 < X < 186)$

$\approx 13.59\%$
 ≈ 0.136

ii $P(X < 162)$

$\approx 1 - (0.5 + 0.3413)$
 ≈ 0.159

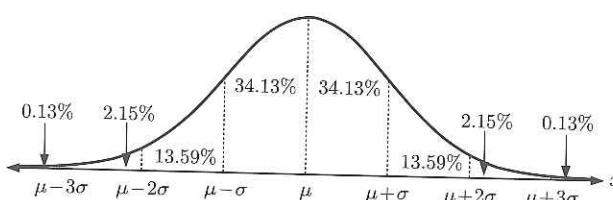
iii $P(X < 154)$

$\approx 0.0215 + 0.0013$
 ≈ 0.0228

iv $P(X > 162)$

$\approx 1 - 0.159 \quad \{\text{using b ii}\}$
 ≈ 0.841

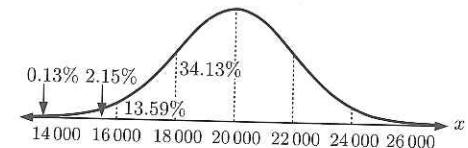
c 16% of students are taller than 178 cm $\{13.59\% + 2.15\% + 0.13\% \approx 16\%\}$
 $\therefore k = 178$

8


a 97.72% of 13 year old boys are taller than 131 cm $\therefore \mu - 2\sigma = 131$
 2.28% of 13 year old boys are taller than 179 cm $\therefore \mu + 2\sigma = 179 \dots (1)$

Adding: $\frac{2\mu}{2} = \frac{179 - 131}{2} = 12$, and so $\mu = 155$ cm
 Substituting $\mu = 155$ cm into (1) gives $\sigma = \frac{179 - 155}{2} = 12$ cm

b For $\mu = 155$ cm and $\sigma = 12$ cm, 143 cm = $\mu - \sigma$, and 191 cm = $\mu + 3\sigma$
 \therefore between 143 cm and 191 cm, the percentage is $34.13\% + 34.13\% + 13.59\% + 2.15\% \approx 84.0\%$
 So the probability is 0.84.

9


b $P(X > 16000)$

$\approx 0.1359 + 0.3413 + 0.5$
 ≈ 0.9772

c $P(18000 \leq X \leq 24000)$
 $\approx 0.3413 \times 2 + 0.1359$
 ≈ 0.8185

\therefore we expect that over 16000 bottles are filled on $260 \times 0.9772 \approx 254$ days.

\therefore we expect that between 18000 and 24000 bottles are filled on $260 \times 0.8185 \approx 213$ days.

EXERCISE 24B.2

1 a 0.341

b 0.383

c 0.106

2 a 0.341

b 0.264

c 0.212

d 0.945

e 0.579

f 0.383

3 a $P(X < a) = 0.378$

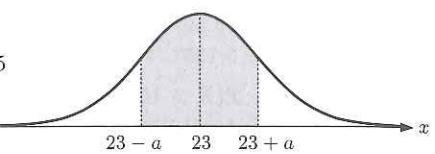
$\therefore a \approx 21.4$

b $P(X \geq a) = 0.592$

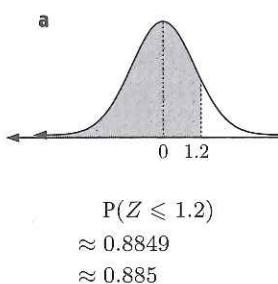
$\therefore P(X < a) = 1 - 0.592 = 0.408$

$\therefore a \approx 21.8$

c $P(23 - a < X < 23 + a) = 0.427$
 $\therefore P(23 < X < 23 + a) = \frac{1}{2}(0.427) = 0.2135$
 $\therefore P(X < 23 + a) = 0.5 + 0.2135 = 0.7135$
 $\therefore 23 + a \approx 25.82$
 $\therefore a \approx 2.82$

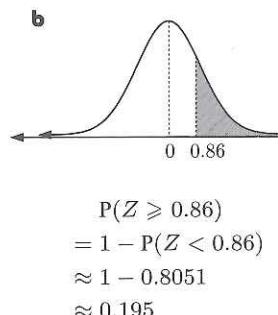

EXERCISE 24C.1

1 a



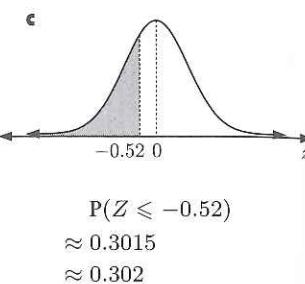
$$\begin{aligned}P(Z \leq 1.2) \\ \approx 0.8849 \\ \approx 0.885\end{aligned}$$

b



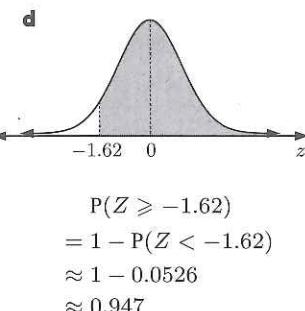
$$\begin{aligned}P(Z \geq 0.86) \\ = 1 - P(Z < 0.86) \\ \approx 1 - 0.8051 \\ \approx 0.195\end{aligned}$$

c



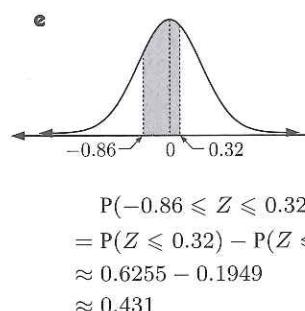
$$\begin{aligned}P(Z \leq -0.52) \\ \approx 0.3015 \\ \approx 0.302\end{aligned}$$

d



$$\begin{aligned}P(Z \geq -1.62) \\ = 1 - P(Z < -1.62) \\ \approx 1 - 0.0526 \\ \approx 0.947\end{aligned}$$

e



$$\begin{aligned}P(-0.86 \leq Z \leq 0.32) \\ = P(Z \leq 0.32) - P(Z \leq -0.86) \\ \approx 0.6255 - 0.1949 \\ \approx 0.431\end{aligned}$$

2 a $P(Z \geq 0.837) \approx 0.201$ b $P(Z \leq 0.0614) \approx 0.524$ c $P(Z \geq -0.876) \approx 0.809$ d $P(-0.3862 \leq Z \leq 0.2506) \approx 0.249$ 3 a $P(-0.5 < Z < 0.5) \approx 0.383$ b $P(-1.960 < Z < 1.960) \approx 0.950$ 4 a $P(Z \leq a) = 0.95$ b $P(Z \geq a) = 0.90$
 $\therefore a \approx 1.645$ {searching in tables
 $\therefore a \approx 1.64$ or using technology}

 $\therefore 1 - P(Z < a) = 0.90$
 $\therefore P(Z < a) = 0.1$

$$\begin{aligned}\therefore a \approx -1.28 - \frac{3}{18}(0.01) \\ \therefore a \approx -1.282 \\ \therefore a \approx -1.28\end{aligned}$$

5 a For Physics, $Z = \frac{83 - 78}{10.8} \approx 0.463$ For Chemistry, $Z = \frac{77 - 72}{11.6} \approx 0.431$ For Maths, $Z = \frac{84 - 74}{10.1} \approx 0.990$ For German, $Z = \frac{91 - 86}{9.6} \approx 0.521$ For Biology, $Z = \frac{72 - 62}{12.2} \approx 0.820$

b Maths, Biology, German, Physics, Chemistry

6 Z-score for algebra = $\frac{56 - 50.2}{15.8} \approx 0.3671$ Z-score for geometry = $\frac{x - 58.7}{18.7}$
 \therefore we need to solve $\frac{x - 58.7}{18.7} = 0.3671$
 $\therefore x - 58.7 \approx 6.86$
 $\therefore x \approx 65.6$ So, Pedro needs a result of 65.6%.

EXERCISE 24C.2
1 X is normal with mean 70, standard deviation 4.

a $P(X \geq 74)$
 $= P\left(\frac{X - 70}{4} \geq \frac{74 - 70}{4}\right)$
 $= P(Z \geq 1)$
 $= 1 - P(Z < 1)$
 $\approx 1 - 0.8413$
 ≈ 0.159

b $P(X \leq 68)$
 $= P\left(\frac{X - 70}{4} \leq \frac{68 - 70}{4}\right)$
 $= P(Z \leq -\frac{1}{2})$
 ≈ 0.309

c $P(60.6 \leq X \leq 68.4)$
 $= P\left(\frac{60.6 - 70}{4} \leq \frac{X - 70}{4} \leq \frac{68.4 - 70}{4}\right)$
 $= P(-2.35 \leq Z \leq -0.4)$
 $\approx 0.3446 - 0.0094$
 ≈ 0.335

2 X is normal with mean 58.3 and standard deviation 8.96.

a $P(X \geq 61.8)$
 $= P\left(\frac{X - 58.3}{8.96} \geq \frac{61.8 - 58.3}{8.96}\right)$
 $= P(Z \geq 0.390625)$
 ≈ 0.348

b $P(X \leq 54.2)$
 $= P\left(\frac{X - 58.3}{8.96} \leq \frac{54.2 - 58.3}{8.96}\right)$
 $\approx P(Z \leq -0.4576)$
 ≈ 0.324

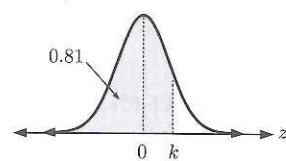
c $P(50.67 \leq X \leq 68.92)$
 $= P\left(\frac{50.67 - 58.3}{8.96} \leq \frac{X - 58.3}{8.96} \leq \frac{68.92 - 58.3}{8.96}\right)$
 $\approx P(-0.85156 \leq Z \leq 1.1853)$
 ≈ 0.685

3 L is normal with mean 50.2 mm and standard deviation 0.93 mm.

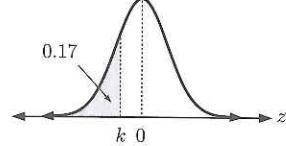
a $P(L \geq 50)$
 $= P\left(\frac{L - 50.2}{0.93} \geq \frac{50 - 50.2}{0.93}\right)$
 $\approx P(Z \geq -0.2151)$
 ≈ 0.585

b $P(L \leq 51)$
 $= P\left(\frac{L - 50.2}{0.93} \leq \frac{51 - 50.2}{0.93}\right)$
 $\approx P(Z \leq 0.8602)$
 ≈ 0.805

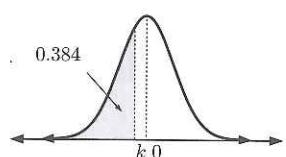
c $P(49 \leq L \leq 50.5)$
 $= P\left(\frac{49 - 50.2}{0.93} \leq \frac{L - 50.2}{0.93} \leq \frac{50.5 - 50.2}{0.93}\right)$
 $= P(-1.2903 \leq Z \leq 0.3226)$
 ≈ 0.528

EXERCISE 24D**1 a**

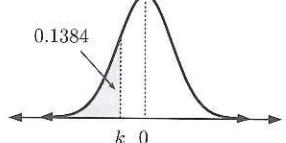
$$\begin{aligned} P(Z \leq k) &= 0.81 \\ \therefore k &\approx 0.87 + \frac{22}{28}(0.01) \\ \therefore k &\approx 0.878 \end{aligned}$$

c

$$\begin{aligned} P(Z \leq k) &= 0.17 \\ \therefore k &\approx -0.96 + \frac{15}{26}(0.01) \\ \therefore k &\approx -0.954 \end{aligned}$$

2 a

$$\begin{aligned} P(Z \leq k) &= 0.384 \\ \therefore k &\approx -0.295 \end{aligned}$$

c

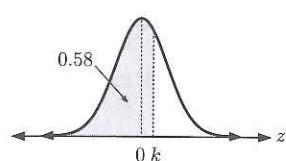
$$\begin{aligned} P(Z \leq k) &= 0.1384 \\ \therefore k &\approx -1.09 \end{aligned}$$

3 a

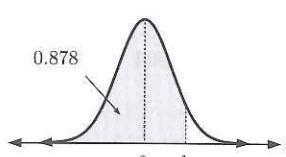
$$\begin{aligned} P(-k \leq Z \leq k) &= P(Z \leq k) - P(Z < -k) \\ &= P(Z \leq k) - P(Z > k) \quad \{ \text{as area 1 = area 2} \} \\ &= P(Z \leq k) - [1 - P(Z \leq k)] \\ &= P(Z \leq k) - 1 + P(Z \leq k) \\ &= 2P(Z \leq k) - 1 \end{aligned}$$

b

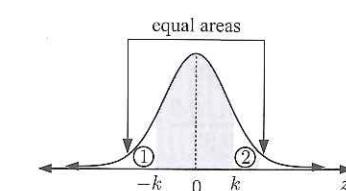
$$\begin{aligned} \text{i} \quad P(-k \leq Z \leq k) &= 0.238 \\ \therefore 2P(Z \leq k) - 1 &= 0.238 \\ \therefore 2P(Z \leq k) &= 1.238 \\ \therefore P(Z \leq k) &= 0.619 \\ \therefore k &\approx 0.303 \end{aligned}$$

b

$$\begin{aligned} P(Z \leq k) &= 0.58 \\ \therefore k &\approx 0.20 + \frac{7}{39}(0.01) \\ \therefore k &\approx 0.202 \end{aligned}$$

b

$$\begin{aligned} P(Z \leq k) &= 0.878 \\ \therefore k &\approx 1.17 \end{aligned}$$



$$\begin{aligned} \text{ii} \quad P(-k \leq Z \leq k) &= 0.7004 \\ \therefore 2P(Z \leq k) - 1 &= 0.7004 \\ \therefore 2P(Z \leq k) &= 1.7004 \\ \therefore P(Z \leq k) &= 0.8502 \\ \therefore k &\approx 1.04 \end{aligned}$$

4 a

$$\begin{aligned} P(X \leq k) &= 0.9 \\ \therefore k &\approx 79.1 \quad \{ \text{using technology} \} \end{aligned}$$

b

$$\begin{aligned} P(X \geq k) &= 0.8 \\ \therefore P(X < k) &= 0.2 \\ \therefore k &\approx 31.3 \quad \{ \text{using technology} \} \end{aligned}$$

EXERCISE 24E**1** Let X be the length of a bolt in cm.

Then X is normally distributed with $\mu = 19.8$ and $\sigma = 0.3$.
 $\therefore P(19.7 < X < 20) \approx 0.378$

2 Let X be the money collected in dollars.

Then X is normally distributed with $\mu = 40$ and $\sigma = 6$.

$$\begin{aligned} \text{a} \quad P(30.00 < X < 50.00) &\approx 0.904 \\ &\approx 90.4\% \end{aligned} \quad \begin{aligned} \text{b} \quad P(X \geq 50) &\approx 0.0478 \\ &\approx 4.78\% \end{aligned}$$

3 Let X be the result of the Physics test.

Then X is normally distributed with $\mu = 46$ and $\sigma = 25$.
 We need to find k such that $P(X \geq k) = 0.07$

$$\begin{aligned} \therefore 1 - P(X < k) &= 0.07 \\ \therefore P(X < k) &= 0.93 \\ \therefore k &\approx 82.894 \\ \therefore k &\approx 83 \quad \{ \text{assuming } k \text{ is an integer} \} \end{aligned}$$

So, the lowest score to get an A would be 83.

4 Let X be the length of an eel in cm.

Then X is normally distributed with $\mu = 41$ and $\sigma = \sqrt{11}$.

$$\begin{aligned} \text{a} \quad P(X \geq 50) &\approx 0.00333 \\ \text{b} \quad P(40 \leq X \leq 50) &\approx 0.615 \\ &\approx 61.5\% \end{aligned} \quad \begin{aligned} \text{c} \quad P(X \geq 45) &\approx 0.114 \\ \text{So, we would expect } 200 \times 0.114 &\approx 23 \text{ eels to be at least 45 cm long.} \end{aligned}$$

5

$$\begin{aligned} P(X \geq 35) &= 0.32 && \text{and} && P(X \leq 8) = 0.26 \\ \therefore P(X < 35) &= 0.68 && \therefore P\left(\frac{X - \mu}{\sigma} \leq \frac{8 - \mu}{\sigma}\right) &= 0.26 \\ \therefore P\left(\frac{X - \mu}{\sigma} < \frac{35 - \mu}{\sigma}\right) &= 0.68 && \therefore P\left(Z \leq \frac{8 - \mu}{\sigma}\right) &= 0.26 \\ \therefore P\left(Z < \frac{35 - \mu}{\sigma}\right) &= 0.68 && \therefore \frac{8 - \mu}{\sigma} &\approx -0.6433 \\ \therefore \frac{35 - \mu}{\sigma} &\approx 0.4677 && \therefore 8 - \mu &\approx -0.6433\sigma \dots (2) \\ \therefore 35 - \mu &\approx 0.4677\sigma \dots (1) \end{aligned}$$

Solving (1) and (2) simultaneously, $35 - 0.4677\sigma \approx 8 + 0.6433\sigma$

$$\begin{aligned} \therefore 27 &\approx 1.111\sigma \\ \therefore \sigma &\approx 24.3 \quad \text{and} \quad \mu = 35 - 0.4677 \times 24.3 \\ &\therefore \mu \approx 23.6 \\ \text{So, } \mu &\approx 23.6 \quad \text{and} \quad \sigma \approx 24.3 \end{aligned}$$

- 6 a** Let the mean be μ and standard deviation be σ .

$$\begin{aligned} \text{Then } P(X \geq 80) &= 0.1 & \text{and } P(X \leq 30) &= 0.15 \\ \therefore P(X < 80) &= 0.9 & \therefore P\left(\frac{X-\mu}{\sigma} \leq \frac{30-\mu}{\sigma}\right) &= 0.15 \\ \therefore P\left(\frac{X-\mu}{\sigma} < \frac{80-\mu}{\sigma}\right) &= 0.9 & \therefore P\left(Z \leq \frac{30-\mu}{\sigma}\right) &= 0.15 \\ \therefore P\left(Z < \frac{80-\mu}{\sigma}\right) &= 0.9 & \therefore \frac{30-\mu}{\sigma} &\approx -1.0364 \\ \therefore \frac{80-\mu}{\sigma} &\approx 1.2816 & \therefore 30-\mu &\approx -1.0364\sigma \dots (2) \\ \therefore 80-\mu &\approx 1.2816\sigma \dots (1) \end{aligned}$$

Solving (1) and (2) simultaneously, $(80-\mu) - (30-\mu) \approx 1.2816\sigma + 1.0364\sigma$

$$\begin{aligned} 50 &\approx 2.318\sigma \\ \therefore \sigma &\approx \frac{50}{2.318} \approx 21.57 \end{aligned}$$

Using (1), $80-\mu \approx 1.2816 \times 21.57 \approx 27.6$

$$\begin{aligned} \therefore \mu &\approx 52.36 \\ \therefore \mu &\approx 52.4 \text{ and } \sigma \approx 21.6 \end{aligned}$$

- b** Let X be the result of the mathematics exam.

X is normally distributed with mean μ and standard deviation σ .

We know that $P(X \geq 80) = 0.1$ and $P(X \leq 30) = 0.15$.

So, from **a**, $\mu \approx 52.36$ and $\sigma \approx 21.57$.

If part marks can be given, $P(X > 50) \approx 0.544$

$$\approx 54.4\%$$

If only integer marks can be given, $P(X \geq 51) \approx 0.525$

$$\approx 52.5\%$$

- 7** Let X be the IQ of a student at the school.

X is normally distributed with mean μ and standard deviation 15.

Now, $P(X \geq 125) = 0.2$

$$\begin{aligned} \therefore P\left(\frac{X-\mu}{15} \geq \frac{125-\mu}{15}\right) &= 0.2 \\ \therefore P\left(Z \geq \frac{125-\mu}{15}\right) &= 0.2 \\ \therefore P\left(Z < \frac{125-\mu}{15}\right) &= 0.8 \\ \therefore \frac{125-\mu}{15} &\approx 0.8416 \\ \therefore \mu &\approx 112 \end{aligned}$$

The mean IQ at the school is 112.

- 8** Let X be the distance jumped by the athlete.

X is normally distributed with mean 5.2 m and standard deviation σ .

Now, $P(X < 5) = 0.15$

$$\begin{aligned} \therefore P\left(\frac{X-5.2}{\sigma} < \frac{5-5.2}{\sigma}\right) &= 0.15 \\ \therefore P\left(Z < -\frac{0.2}{\sigma}\right) &= 0.15 \\ \therefore -\frac{0.2}{\sigma} &\approx -1.036 \\ \therefore \sigma &\approx 0.193 \text{ m} \end{aligned}$$

- 9 a** Let the mean be μ and standard deviation be σ and X be the diameter in cm.

$$\begin{aligned} \therefore P(X < 1.94) &= 0.02 & \text{and } P(X > 2.06) &= 0.03 \\ \therefore P\left(\frac{X-\mu}{\sigma} < \frac{1.94-\mu}{\sigma}\right) &= 0.02 & \therefore P\left(\frac{X-\mu}{\sigma} > \frac{2.06-\mu}{\sigma}\right) &= 0.03 \\ \therefore P\left(Z < \frac{1.94-\mu}{\sigma}\right) &= 0.02 & \therefore P\left(Z > \frac{2.06-\mu}{\sigma}\right) &= 0.03 \\ \therefore \frac{1.94-\mu}{\sigma} &\approx -2.054 & \therefore P\left(Z \leq \frac{2.06-\mu}{\sigma}\right) &= 0.97 \\ \therefore 1.94-\mu &\approx -2.054\sigma \dots (1) & \therefore \frac{2.06-\mu}{\sigma} &\approx 1.881 \\ \therefore \frac{2.06-\mu}{\sigma} &\approx 1.881 & 2.06-\mu &\approx 1.881\sigma \dots (2) \end{aligned}$$

Solving (1) and (2) simultaneously, $(2.06-\mu) - (1.94-\mu) = 1.881\sigma + 2.054\sigma$

$$\begin{aligned} \therefore 3.935\sigma &= 0.12 \\ \therefore \sigma &\approx 0.0305 \end{aligned}$$

Using (1), $1.94-\mu \approx -2.054 \times 0.0305 \approx -0.0626$

$$\therefore \mu \approx 2.00$$

$$\therefore \mu \approx 2.00 \text{ and } \sigma \approx 0.0305$$

- b** Let Y be the number of tokens which will not operate the machine. This is a binomial situation with the probability $p = 0.02 + 0.03 = 0.05$ of failure to operate and $n = 20$. So, $Y \sim B(20, 0.05)$.

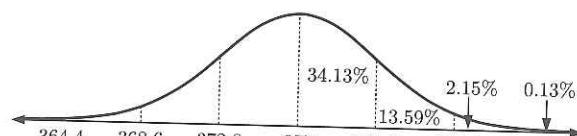
$$\begin{aligned} \therefore P(\text{at most one will not operate}) &= P(Y \leq 1) \\ &\approx 0.736 \end{aligned}$$

REVIEW SET 24A

- 1** X is the contents of the container in mL.

X is normally distributed with $\mu = 377$ and $\sigma = 4.2$.

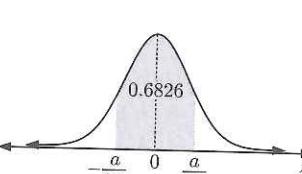
$$\begin{aligned} \text{a i } P(X < 368.6) & \approx 2.15\% + 0.13\% \\ & \approx 2.28\% \\ \text{ii } P(372.8 < X < 389.6) & \approx 2 \times 34.13\% + 13.59\% + 2.15\% \\ & \approx 84.0\% \end{aligned}$$

$$\begin{aligned} \text{b } P(377 < X < 381.2) & \approx 0.341 \end{aligned}$$


- 2** Let X denote the mass of a Coffin Bay Oyster. X is distributed normally with a mean of 38.6 and a standard deviation of 6.3.

$$\begin{aligned} \text{a } P(38.6 - a \leq X \leq 38.6 + a) &= 0.6826 \\ \therefore P\left(\frac{38.6-a-38.6}{6.3} \leq \frac{X-38.6}{6.3} \leq \frac{38.6+a-38.6}{6.3}\right) &= 0.6826 \end{aligned}$$

$$\begin{aligned} \therefore P\left(-\frac{a}{6.3} \leq Z \leq \frac{a}{6.3}\right) &= 0.6826 \\ \therefore \text{by symmetry, } P\left(Z \leq -\frac{a}{6.3}\right) &= \frac{1-0.6826}{2} \\ \therefore P\left(Z \leq -\frac{a}{6.3}\right) &= 0.1587 \dots (*) \end{aligned}$$

$$\begin{aligned} \therefore -\frac{a}{6.3} &\approx -1.00 \\ \therefore a &\approx 6.30 \text{ g} \end{aligned}$$


b

$$\begin{aligned} P(X \geq b) &= 0.8413 \\ \therefore P(X < b) &= 0.1587 \\ \therefore P\left(\frac{X - 38.6}{6.3} < \frac{b - 38.6}{6.3}\right) &= 0.1587 \end{aligned}$$

$$\text{Comparing with } (*), \frac{b - 38.6}{6.3} = -\frac{a}{6.3}$$

$$\therefore b - 38.6 \approx -6.30$$

$$\therefore b \approx 32.3 \text{ g}$$

3 a

$$\int_0^2 ax(x-3) dx = 1$$

$$\therefore a \int_0^2 (x^2 - 3x) dx = 1$$

$$\therefore a \left[\frac{1}{3}x^3 - \frac{3}{2}x^2 \right]_0^2 = 1$$

$$\therefore a \left[\frac{8}{3} - 6 \right] = 1$$

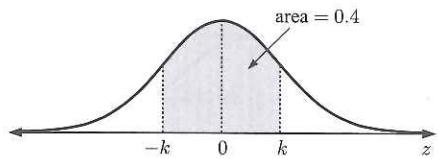
$$\therefore a \left(-\frac{10}{3} \right) = 1$$

$$\therefore a = -\frac{3}{10}$$

c

$$\begin{aligned} \mu &= \int_0^2 x f(x) dx \\ &= \int_0^2 -\frac{3}{10}x^2(x-3) dx \\ &= -\frac{3}{10} \int_0^2 (x^3 - 3x^2) dx \\ &= -\frac{3}{10} \left[\frac{1}{4}x^4 - x^3 \right]_0^2 \\ &= -\frac{3}{10} \left(\frac{1}{4}(16) - 8 \right) dx \\ &= -\frac{3}{10}(-4) \\ &= \frac{6}{5} = 1.2 \end{aligned}$$

4 $P(-k \leq Z \leq k) = 0.4$



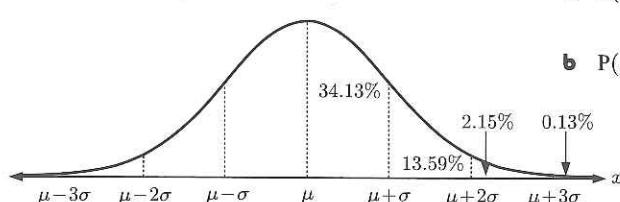
5 Jarrod's z-score is $\frac{41 - 35}{4} = 1.5$

$$\therefore \text{Paul needs } x \text{ such that } \frac{x - 25}{3} = 1.5$$

$$\therefore x = 25 + 4.5 = 29.5$$

Paul needs to throw a tennis ball 29.5 m to perform as well as Jarrod.

6



a $P(\mu + \sigma < X < \mu + 2\sigma) \approx 13.59\%$
 ≈ 0.136

b $P(\mu - \sigma < X < \mu + \sigma) \approx 34.13\%$
 ≈ 0.341

REVIEW SET 24B

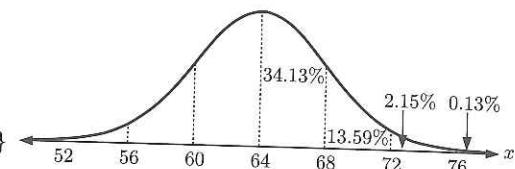
- 1 If random variable X is the arm length in cm then X is normally distributed with $\mu = 64$ and $\sigma = 4$.

a i $P(60 < X < 72)$
 $\approx 2 \times 34.13\% + 13.59\%$
 $\approx 81.9\%$

ii $P(X > 60)$
 $\approx 50\% + 34.13\%$
 $\approx 84.1\%$

b $P(56 < X < 64) \approx 0.3413 + 0.1359$
 ≈ 0.477

c $P(X > x) = 0.7$
 $\therefore P(X \leq x) = 0.3$
 $\therefore x \approx 61.9$ {using technology}



- 2 Let X be the rod length in mm.

X is normally distributed with mean μ and $\sigma = 3$.

Now $P(X < 25) = 0.02$

 $\therefore P\left(\frac{X - \mu}{3} < \frac{25 - \mu}{3}\right) = 0.02$
 $\therefore \frac{25 - \mu}{3} \approx -2.0537$
 $\therefore P\left(Z < \frac{25 - \mu}{3}\right) = 0.02$
 $\therefore 25 - \mu \approx -6.161$
 $\therefore \mu \approx 31.2$

\therefore the mean rod length is 31.2 mm.

- 3 a Since Area A = Area B , 20 and 38 must be equal distances away from the mean μ , because of the symmetry of the normal distribution.

$$\therefore \mu \text{ is halfway between 20 and 38, so } \mu = \frac{20 + 38}{2} = 29.$$

Now $P(X \leq 20) = 0.2$

 $\therefore P\left(\frac{X - 29}{\sigma} \leq \frac{20 - 29}{\sigma}\right) = 0.2$
 $\therefore P\left(Z \leq -\frac{9}{\sigma}\right) = 0.2$
 $\therefore -\frac{9}{\sigma} \approx -0.8416$
 $\therefore \sigma \approx 10.69$

$\therefore \mu = 29, \sigma \approx 10.69$

b Using the values obtained for μ and σ in a and technology:

i $P(X \leq 35) \approx 0.713$

ii $P(23 \leq X \leq 30) \approx 0.250$

- 4 Let X be the marks in the examination. Then X is normally distributed with $\mu = 49$ and $\sigma = 15$.

a $P(X \geq 45) \approx 0.6051$
 So, $2376 \times 0.6051 \approx 1438$ students passed the examination.

b Let k be the minimum mark required for a '7'.
 $\therefore P(X \geq k) = 0.07$

$$\therefore P(X < k) = 1 - 0.07 = 0.93$$

$$\therefore k \approx 71.1$$

$\therefore k \approx 71$ (to the nearest integer)

So the minimum mark required to obtain a '7' is 71 marks.

5 X is the life of a battery in weeks.

X is normally distributed with $\mu = 33.2$ and $\sigma = 2.8$.

a $P(X \geq 35) \approx 0.260$

b We need to find k such that $P(X \leq k) = 0.08$
 $\therefore k \approx 29.3$

So, the manufacturer can expect the batteries to last 29.3 weeks before 8% of them fail.

6 a $P(X \leq 30) = 0.0832$ and $P(X \geq 90) = 0.101$
 $\therefore P\left(\frac{X-\mu}{\sigma} \leq \frac{30-\mu}{\sigma}\right) \approx 0.0832$ $\therefore P(X < 90) = 0.899$
 $\therefore P\left(Z \leq \frac{30-\mu}{\sigma}\right) \approx 0.0832$ $\therefore P\left(\frac{X-\mu}{\sigma} < \frac{90-\mu}{\sigma}\right) = 0.899$
 $\therefore \frac{30-\mu}{\sigma} \approx -1.383864$ $\therefore P\left(Z < \frac{90-\mu}{\sigma}\right) = 0.899$
 $\therefore 30-\mu \approx -1.383864\sigma \dots (1)$ $\therefore \frac{90-\mu}{\sigma} \approx 1.275874$
 $\therefore 90-\mu = 1.275874\sigma \dots (2)$

Solving (1) and (2) simultaneously,

$$(90-\mu) - (30-\mu) \approx 1.275874\sigma - (-1.383864\sigma) \\ \therefore 60 \approx 2.6597\sigma \\ \therefore \sigma \approx 22.559$$

Using (2), $90-\mu \approx 1.275874(22.559)$

$$\therefore \mu \approx 90 - 1.275874(22.559) \\ \approx 61.218$$

b $P(-7 \leq X - \mu \leq 7) \approx P(-7 \leq X - 61.218 \leq 7)$
 $\approx P(54.218 \leq X \leq 68.218)$
 ≈ 0.244

REVIEW SET 24C

1 Using technology:

a $P(X \geq 22) \approx 0.364$

b $P(18 \leq X \leq 22) \approx 0.356$

c $P(X \leq k) = 0.3$

$\therefore k \approx 18.2$

2 Let X be the volume of drink in mL.

Then X is normally distributed with $\mu = 376$.

Now $P(X < 375) = 0.023$

$$\therefore P\left(\frac{X-376}{\sigma} < \frac{375-376}{\sigma}\right) = 0.023 \\ \therefore P\left(Z < \frac{-1}{\sigma}\right) = 0.023 \\ \therefore -\frac{1}{\sigma} \approx -1.995 \\ \therefore \sigma \approx 0.501$$

\therefore the standard deviation is 0.501 mL

3 $P(-0.524 < X - \mu < 0.524) = P\left(\frac{-0.524}{2} < \frac{X-\mu}{2} < \frac{0.524}{2}\right)$
 $= P(-0.262 < Z < 0.262)$
 $\approx 0.207 \quad \{\text{using technology}\}$

4 Let X be the length of the rods. X is normally distributed with $\sigma = 6$.

Now $P(X \geq 89.52) = 0.0563$

$$\therefore P(X < 89.52) = 1 - 0.0563$$

$$\therefore P\left(\frac{X-\mu}{\sigma} < \frac{89.52-\mu}{\sigma}\right) = 0.9437$$

$$\therefore P\left(Z < \frac{89.52-\mu}{6}\right) = 0.9437$$

$$\therefore \frac{89.52-\mu}{6} \approx 1.5866$$

$$\therefore 89.52 - \mu \approx 9.52$$

$$\therefore \mu \approx 80.0$$

So, the mean is 80.0 cm.

5 $P(X < 90) \approx 0.975$

$$\therefore P\left(\frac{X-50}{\sigma} < \frac{90-50}{\sigma}\right) \approx 0.975$$

$$\therefore P\left(Z < \frac{40}{\sigma}\right) \approx 0.975$$

$$\therefore \frac{40}{\sigma} \approx 1.95996$$

$$\therefore \sigma \approx 20.409$$

So, $X \sim N(50, 20.409^2)$

Now, the shaded area = $P(X \geq 80)$
 $\approx 0.0708 \text{ units}^2$

6 Let X be the heights of 18 year old boys. X is normally distributed with $\mu = 187$.

Now $P(X > 193) = 0.15$

$$\therefore P(X \leq 193) = 0.85$$

$$\therefore P\left(\frac{X-187}{\sigma} \leq \frac{193-187}{\sigma}\right) = 0.85$$

$$\therefore P\left(Z \leq \frac{6}{\sigma}\right) = 0.85$$

$$\therefore \frac{6}{\sigma} \approx 1.0364$$

$$\therefore \sigma \approx 5.789$$

So, $P(X > 185) \approx 0.635$

the probability that two 18 year old boys are taller than 185 cm $\approx 0.635^2$
 ≈ 0.403