Sequences

Chapter 26 Supplement

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12.2 ARITHMETIC SEQUENCES

OBJECTIVES
1. Determine If a Sequence Is Arithmetic
2. Find a Formula for an Arithmetic Sequence
3. Find the Sum of an Arithmetic Sequence

1 When the difference between successive terms of a sequence is always the same number, the sequence is called **arithmetic.** Thus, an **arithmetic sequence*** may be defined recursively as $a_1 = a$, $a_n - a_{n-1} = d$, or as

$$a_1 = a, \qquad a_n = a_{n-1} + d \tag{1}$$

where $a = a_1$ and d are real numbers. The number a is the first term, and the number d is called the **common difference.**

The terms of an arithmetic sequence with first term a and common difference d follow the pattern

$$a, \quad a + d, \quad a + 2d, \quad a + 3d, \ldots$$

EXAMPLE 1 Determining If a Sequence Is Arithmetic

The sequence

$$4, 7, 10, 13, \ldots$$

is arithmetic since the difference of successive terms is 3. The first term is 4, and the common difference is 3.

EXAMPLE 2 Determining If a Sequence Is Arithmetic

Show that the following sequence is arithmetic. Find the first term and the common difference.

$$\{s_n\} = \{3n + 5\}$$

Solution The first term is $s_1 = 3 \cdot 1 + 5 = 8$. The nth and $(n - 1)$st terms of the sequence $\{s_n\}$ are

$$s_n = 3n + 5 \quad \text{and} \quad s_{n-1} = 3(n - 1) + 5 = 3n + 2$$

Their difference is

$$s_n - s_{n-1} = (3n + 5) - (3n + 2) = 5 - 2 = 3$$

Since the difference of two successive terms does not depend on n, the common difference is 3 and the sequence is arithmetic.

EXAMPLE 3 Determining If a Sequence Is Arithmetic

Show that the sequence $\{t_n\} = \{4 - n\}$ is arithmetic. Find the first term and the common difference.

* Sometimes called an **arithmetic progression.**

Solution The first term is $t_1 = 4 - 1 = 3$. The nth and $(n-1)$st terms are

$$t_n = 4 - n \quad \text{and} \quad t_{n-1} = 4 - (n-1) = 5 - n$$

Their difference is

$$t_n - t_{n-1} = (4 - n) - (5 - n) = 4 - 5 = -1$$

The difference of two successive terms does not depend on n; it always equals the same number, -1. Hence, $\{t_n\}$ is an arithmetic sequence whose common difference is -1.

NOW WORK PROBLEM 3.

2 Suppose that a is the first term of an arithmetic sequence whose common difference is d. We seek a formula for the nth term, a_n. To see the pattern, we write down the first few terms:

$$\begin{aligned}
a_1 &= a \\
a_2 &= a_1 + d = a + 1 \cdot d \\
a_3 &= a_2 + d = (a + d) + d = a + 2 \cdot d \\
a_4 &= a_3 + d = (a + 2 \cdot d) + d = a + 3 \cdot d \\
a_5 &= a_4 + d = (a + 3 \cdot d) + d = a + 4 \cdot d \\
&\vdots \\
a_n &= a_{n-1} + d = [a + (n-2)d] + d = a + (n-1)d
\end{aligned}$$

We are led to the following result:

Theorem **nth Term of an Arithmetic Sequence**

For an arithmetic sequence $\{a_n\}$ whose first term is a and whose common difference is d, the nth term is determined by the formula

$$\boxed{a_n = a + (n-1)d \qquad (2)}$$

EXAMPLE 4 Finding a Particular Term of an Arithmetic Sequence

Find the thirteenth term of the arithmetic sequence: $2, 6, 10, 14, 18, \ldots$

Solution The first term of this arithmetic sequence is $a = 2$, and the common difference is 4. By formula (2), the nth term is

$$a_n = 2 + (n-1)4$$

Hence, the thirteenth term is

$$a_{13} = 2 + 12 \cdot 4 = 50$$

EXPLORATION Use a graphing utility to find the thirteenth term of the sequence given in Example 4. Use it to find the twentieth and fiftieth terms.

EXAMPLE 5 Finding a Recursive Formula for an Arithmetic Sequence

The eighth term of an arithmetic sequence is 75, and the twentieth term is 39. Find the first term and the common difference. Give a recursive formula for the sequence.

Solution By formula (2), we know that $a_n = a + (n - 1)d$. As a result,

$$\begin{cases} a_8 = a + 7d = 75 \\ a_{20} = a + 19d = 39 \end{cases}$$

This is a system of two linear equations containing two variables, a and d, which we can solve by elimination. Subtracting the second equation from the first equation, we get

$$-12d = 36$$

$$d = -3$$

With $d = -3$, we find that $a = 75 - 7d = 75 - 7(-3) = 96$. The first term is $a = 96$, and the common difference is $d = -3$. A recursive formula for this sequence is found using formula (1).

$$a_1 = 96, \qquad a_n = a_{n-1} - 3$$

Based on formula (2), a formula for the nth term of the sequence $\{a_n\}$ in Example 5 is

$$a_n = a + (n - 1)d = 96 + (n - 1)(-3) = 99 - 3n$$

NOW WORK PROBLEMS 19 AND 25.

EXPLORATION Graph the recursive formula from Example 5, $a_1 = 96$, $a_n = a_{n-1} - 3$, using a graphing utility. Conclude that the graph of the recursive formula behaves like the graph of a linear function. How is d, the common difference, related to m, the slope of a line?

ADDING THE FIRST n TERMS OF AN ARITHMETIC SEQUENCE

3 The next result gives a formula for finding the sum of the first n terms of an arithmetic sequence.

Theorem **Sum of n Terms of an Arithmetic Sequence**

Let $\{a_n\}$ be an arithmetic sequence with first term a and common difference d. The sum S_n of the first n terms of $\{a_n\}$ is

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + a_n) \qquad (3)$$

Proof

$$
\begin{aligned}
S_n &= a_1 + a_2 + a_3 + \cdots + a_n && \text{Sum of first n terms} \\
&= a + (a + d) + (a + 2d) + \cdots + [a + (n-1)d] && \text{Formula (2)} \\
&= \underbrace{(a + a + \cdots + a)}_{n \text{ terms}} + [d + 2d + \cdots + (n-1)d] && \text{Rearrange terms} \\
&= na + d[1 + 2 + \cdots + (n-1)] \\
&= na + d\left[\frac{(n-1)n}{2}\right] && \text{Property 6, Section 12.1} \\
&= na + \frac{n}{2}(n-1)d \\
&= \frac{n}{2}[2a + (n-1)d] && \text{Factor out n/2} \qquad (4) \\
&= \frac{n}{2}[a + a + (n-1)d] \\
&= \frac{n}{2}(a + a_n) && \text{Formula (2)} \qquad (5)
\end{aligned}
$$

■

Formula (3) provides two ways to find the sum of the first n terms of an arithmetic sequence. Notice that (4) involves the first term and common difference, while (5) involves the first term and the nth term. Use whichever form is easier.

EXAMPLE 6 Finding the Sum of n Terms of an Arithmetic Sequence

Find the sum S_n of the first n terms of the sequence $\{3n + 5\}$; that is, find

$$8 + 11 + 14 + \cdots + (3n + 5)$$

Solution The sequence $\{3n + 5\}$ is an arithmetic sequence with first term $a = 8$ and the nth term $(3n + 5)$. To find the sum S_n, we use formula (3), as given in (5).

$$S_n = \frac{n}{2}(a + a_n) = \frac{n}{2}[8 + (3n + 5)] = \frac{n}{2}(3n + 13)$$ ■

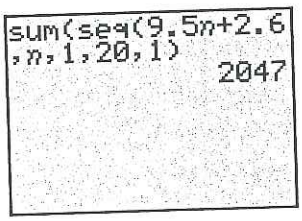
NOW WORK PROBLEM 33.

EXAMPLE 7 Using a Graphing Utility to Find the Sum of 20 Terms of an Arithmetic Sequence

Use a graphing utility to find the sum S_n of the first 20 terms of the sequence $\{9.5n + 2.6\}$.

Solution Figure 12 shows the results obtained using a TI-83 graphing calculator.

Figure 12



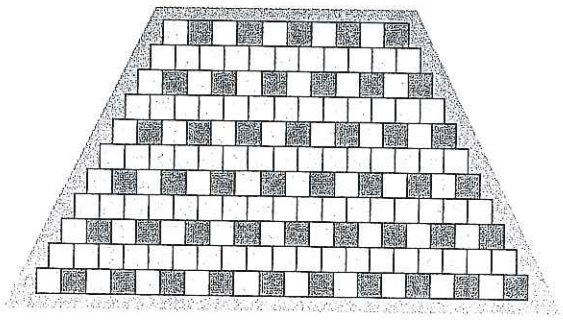
The sum of the first 20 terms of the sequence $\{9.5n + 2.6\}$ is 2047. ■

NOW WORK PROBLEM 41.

EXAMPLE 8 Creating a Floor Design

A ceramic tile floor is designed in the shape of a trapezoid 20 feet wide at the base and 10 feet wide at the top. See Figure 13. The tiles, 12 inches by 12 inches, are to be placed so that each successive row contains one less tile than the preceding row. How many tiles will be required?

Figure 13



Solution The bottom row requires 20 tiles and the top row, 10 tiles. Since each successive row requires one less tile, the total number of tiles required is

$$S = 20 + 19 + 18 + \cdots + 11 + 10$$

This is the sum of an arithmetic sequence; the common difference is -1. The number of terms to be added is $n = 11$, with the first term $a = 20$ and the last term $a_{11} = 10$. The sum S is

$$S = \frac{n}{2}(a + a_{11}) = \frac{11}{2}(20 + 10) = 165$$

Thus, 165 tiles will be required.

12.2 EXERCISES

Problems 1–10, an arithmetic sequence is given. Find the common difference and write out the first four terms.

1. $\{n + 4\}$
2. $\{n - 5\}$
3. $\{2n - 5\}$
4. $\{3n + 1\}$
5. $\{6 - 2n\}$
6. $\{4 - 2n\}$
7. $\left\{\frac{1}{2} - \frac{1}{3}n\right\}$
8. $\left\{\frac{2}{3} + \frac{n}{4}\right\}$
9. $\{\ln 3^n\}$
10. $\{e^{\ln n}\}$

In Problems 11–18, find the nth term of the arithmetic sequence whose initial term a and common difference d are given. What is the fifth term?

11. $a = 2;\quad d = 3$
12. $a = -2;\quad d = 4$
13. $a = 5;\quad d = -3$
14. $a = 6;\quad d = -2$
15. $a = 0;\quad d = \frac{1}{2}$
16. $a = 1;\quad d = -\frac{1}{3}$
17. $a = \sqrt{2};\quad d = \sqrt{2}$
18. $a = 0;\quad d = \pi$

In Problems 19–24, find the indicated term in each arithmetic sequence.

19. 12th term of $2, 4, 6, \ldots$
20. 8th term of $-1, 1, 3, \ldots$
21. 10th term of $1, -2, -5, \ldots$
22. 9th term of $5, 0, -5, \ldots$
23. 8th term of $a, a + b, a + 2b, \ldots$
24. 7th term of $2\sqrt{5}, 4\sqrt{5}, 6\sqrt{5}, \ldots$

In Problems 25–32, find the first term and the common difference of the arithmetic sequence described. Give a recursive formula for the sequence.

25. 8th term is 8; 20th term is 44
26. 4th term is 3; 20th term is 35
27. 9th term is -5; 15th term is 31
28. 8th term is 4; 18th term is -96

29. 15th term is 0; 40th term is -50

30. 5th term is -2; 13th term is 30

31. 14th term is -1; 18th term is -9

32. 12th term is 4; 18th term is 28

In Problems 33–40, find the sum.

33. $1 + 3 + 5 + \cdots + (2n - 1)$

34. $2 + 4 + 6 + \cdots + 2n$

35. $7 + 12 + 17 + \cdots + (2 + 5n)$

36. $-1 + 3 + 7 + \cdots + (4n - 5)$

37. $2 + 4 + 6 + \cdots + 70$

38. $1 + 3 + 5 + \cdots + 59$

39. $5 + 9 + 13 + \cdots + 49$

40. $2 + 5 + 8 + \cdots + 41$

For Problems 41–46, use a graphing utility to find the sum of each sequence.

41. $\{3.45n + 4.12\}, \quad n = 20$

42. $\{2.67n - 1.23\}, \quad n = 25$

43. $2.8 + 5.2 + 7.6 + \cdots + 36.4$

44. $5.4 + 7.3 + 9.2 + \cdots + 32$

45. $4.9 + 7.48 + 10.06 + \cdots + 66.82$

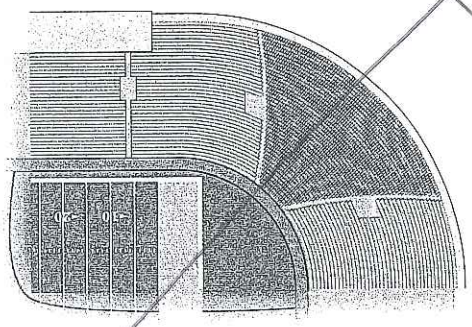
46. $3.71 + 6.9 + 10.09 + \cdots + 80.27$

47. Find x so that $x + 3, 2x + 1$, and $5x + 2$ are consecutive terms of an arithmetic sequence.

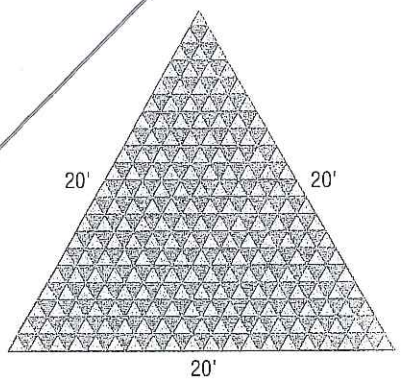
48. Find x so that $2x, 3x + 2$, and $5x + 3$ are consecutive terms of an arithmetic sequence.

49. Drury Lane Theater The Drury Lane Theater has 25 seats in the first row and 30 rows in all. Each successive row contains one additional seat. How many seats are in the theater?

50. Football Stadium The corner section of a football stadium has 15 seats in the first row and 40 rows in all. Each successive row contains two additional seats. How many seats are in this section?



51. Creating a Mosaic A mosaic is designed in the shape of an equilateral triangle, 20 feet on each side. Each tile in the mosaic is in the shape of an equilateral triangle, 12 inches to a side. The tiles are to alternate in color as shown in the illustration. How many tiles of each color will be required?



52. Constructing a Brick Staircase A brick staircase h total of 30 steps. The bottom step requires 100 br Each successive step requires two less bricks than prior step.
(a) How many bricks are required for the top step?
(b) How many bricks are required to build the stairc

53. Stadium Construction How many rows are in the co section of a stadium containing 2040 seats if the first has 10 seats and each successive row has 4 additi seats?

54. Salary Suppose that you just received a job offer w starting salary of $35,000 per year and a guaranteed i of $1400 per year. How many years will it take be your aggregate salary is $280,000?
[**Hint:** Your aggregate salary after two years is $35,0 ($35,000 + $1400).]

55. Make up an arithmetic sequence. Give it to a friend ask for its twentieth term.

11 Geometric sequences

Objective To find a formula for the nth term of a geometric sequence and to find specified terms of geometric sequences.

The method for finding the nth term of a geometric sequence is similar to the method used for arithmetic sequences. For example, to find the tenth term of the geometric sequences 5, 10, 20, 40, . . . , look at the number of times the first term 5 is multiplied by the common ratio 2 to produce t_{10}.

sequence 5 10 20 40 . . . t_{10}

ratios 2 2 2 2 . . . 2

9 ratios

As the diagram shows, the number of ratios is always one less than the number of terms. Therefore, the tenth term is the product of 5 and the ninth power of 2.

$$\begin{aligned} t_{10} &= 5 \cdot 2^9 \\ &= 2560 \end{aligned}$$

Example 1 Find a formula for the nth term of the sequence 3, −12, 48, −192, ...

Solution The first term is $t_1 = 3$. The common ratio is $r = \frac{-12}{3} = -4$.

Since there are n terms, start with 3 and multiply by the ratio $n - 1$ times:

$$t_n = 3(-4)^{n-1} \quad \textit{Answer}$$

The method used in Example 1 can be generalized to *any* geometric sequence.

In a geometric sequence with first term t_1 and common ratio r, the nth (or *general*) term is given by

$$t_n = t_1 \cdot r^{n-1}.$$

Example 2 Find t_6 for the following geometric sequence:

3, −12, 48, . . .

Solution Use the formula $t_n = 3(-4)^{n-1}$ from Example 1.

$$\begin{aligned} t_6 &= 3(-4)^{6-1} \\ &= 3(-4)^5 \\ &= 3(-1024) = -3072 \quad \textit{Answer} \end{aligned}$$

Example 3 Find t_7 for the geometric sequence in which $t_2 = 24$ and $t_5 = 3$.

Solution Substitute 24 for t_2 and 3 for t_5 in the formula $t_n = t_1 \cdot r^{n-1}$ to obtain a system of equations in t_1 and r.

$$t_2 = t_1 \cdot r^{2-1} \longrightarrow 24 = t_1 \cdot r$$
$$t_5 = t_1 \cdot r^{5-1} \longrightarrow 3 = t_1 \cdot r^4$$

Solve the first equation for t_1: $t_1 = \frac{24}{r}$. Substitute this expression for t_1 in the second equation and solve for r:

$$3 = \frac{24}{r} \cdot r^4$$
$$\frac{1}{8} = r^3, \quad \text{so } r = \frac{1}{2}$$

Substitute $\frac{1}{2}$ for r in the first equation:

$$24 = t_1 \cdot \frac{1}{2}$$
$$48 = t_1$$

Using $t_1 = 48$ and $r = \frac{1}{2}$ in the formula $t_n = t_1 \cdot r^{n-1}$, find t_7:

$$t_7 = 48 \cdot \left(\frac{1}{2}\right)^{7-1}$$
$$= 48 \cdot \frac{1}{64} = \frac{3}{4} \quad \textit{Answer}$$

Geometric means are the terms between two given terms of a geometric sequence.

$1, \underbrace{2, 4, 8}, 16$	$1, \underbrace{-2, 4, -8}, 16$	$1, \underbrace{4}, 16$
Three geometric means between 1 and 16	Three geometric means between 1 and 16	*The* geometric mean of 1 and 16

Because 1, 4, 16 and 1, −4, 16 are both geometric sequences, you might think that 4 or −4 is the geometric mean of 1 and 16. However, so that the geometric mean of two numbers will be unique, it is a common practice to consider *the* **geometric mean** of a and b to be $\sqrt{ab}$ if a and b are positive and $-\sqrt{ab}$ if a and b are negative.

Example 4 **a.** Find the geometric mean of 4 and 9.

b. Insert three geometric means between $\frac{1}{2}$ and $\frac{1}{162}$.

Solution **a.** The geometric mean of 4 and 9 is $\sqrt{4 \cdot 9} = \sqrt{36}$, or 6. *Answer*

b. Outline the sequence: $\frac{1}{2}$, ___?___, ___?___, ___?___, $\frac{1}{162}$

(Solution continues on the next page.)

To find r, substitute $\frac{1}{162}$ for t_5 and $\frac{1}{2}$ for t_1 in the formula $t_n = t \cdot r^{n-1}$:

$$\frac{1}{162} = \frac{1}{2} \cdot r^{5-1}$$
$$\frac{1}{81} = r^4$$
$$\pm\frac{1}{3} = r$$

If $r = \frac{1}{3}$, then the geometric sequence is $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54}, \frac{1}{162}$.

If $r = -\frac{1}{3}$, then the geometric sequence is $\frac{1}{2}, -\frac{1}{6}, \frac{1}{18}, -\frac{1}{54}, \frac{1}{162}$.

Answer

Example 5 George has taken a job with a starting salary of \$20,000. Find his salary during his fourth year on the job if he receives annual raises of:

a. \$1100 **b.** 5%

Solution

a. With annual raises of \$1100, George's yearly salaries form an *arithmetic* sequence with $t_1 = 20{,}000$ and $d = 1100$.
The fourth term in the sequence is:

$$\begin{aligned} t_4 &= 20{,}000 + (4 - 1)1100 \\ &= 20{,}000 + 3300 \\ &= 23{,}300 \end{aligned}$$

$\therefore$ George's fourth-year salary will be \$23,300. ***Answer***

b. With annual raises of 5%, George's yearly salaries form a *geometric* sequence with $t_1 = 20{,}000$ and $r = 1.05$. (Notice that the common ratio is 1.05 and not 0.05, because a 5% raise means that for a salary S one year, the salary will be $S + 0.05S$, or $1.05S$, the next year.)
The fourth term in the sequence is:

$$\begin{aligned} t_4 &= 20{,}000(1.05)^{4-1} \\ &= 20{,}000(1.157625) \\ &= 23{,}152.50 \end{aligned}$$

$\therefore$ George's fourth-year salary will be \$23,152.50. ***Answer***

Oral Exercises

Give a formula for the *n*th term of each geometric sequence.

1. 1000, 200, 40, 8, . . .

2. 3, −6, 12, −24, . . .

3. 4, −12, 36, −108, . . .

4. 125, 25, 5, 1, . . .

For each geometric sequence, find the value of t_{11} (in exponential form).

5. 1, 4, 16, 64, . . .
6. 5, 10, 20, 40, . . .
7. 18, −6, 2, $-\frac{2}{3}$, . . .
8. Give the geometric mean of each pair of numbers.
   a. 1, 4
   b. −3, −27
   c. 3, 12

Written Exercises

Find a formula for the *n*th term of each geometric sequence.

A
1. 2, 6, 18, 54, . . .
2. 500, 100, 20, 4, . . .
3. 1, $\sqrt{2}$, 2, $2\sqrt{2}$, . . .
4. 8, 12, 18, 27, . . .
5. 64, −48, 36, −27, . . .
6. −1, 0.1, −0.01, 0.001, . . .

Find the specified term of each geometric sequence.

7. 2, 6, 18, 54, . . . ; t_{10}
8. 5, 10, 20, 40, . . . ; t_{12}
9. 320, 80, 20, 5, . . . ; t_8
10. 1, −3, 9, −27, . . . ; t_8
11. 40, −20, 10, −5, . . . ; t_{11}
12. −10, 50, −250, 1250, . . . ; t_9
13. $t_2 = 18$, $t_3 = 12$; t_5
14. $t_3 = -12$, $t_6 = 96$; t_9
15. $t_1 = 5$, $t_3 = 80$; t_6
16. $t_2 = 8$, $t_4 = 72$; t_1
17. $y, y^3, y^5, \ldots$; t_{20}
18. $ab^2, a^2b^5, a^3b^8, \ldots$; t_{25}

Find the geometric mean of each pair of numbers.

19. 2, 8
20. $\frac{1}{12}, \frac{1}{18}$
21. $\sqrt{3}, 3\sqrt{3}$
22. −18, −36

Insert the given number of geometric means between the pairs of numbers.

23. Three; 5, 80
24. Two; −4, 108
25. Four; 1, 2
26. Three; $\frac{1}{5}, \frac{5}{4}$

Tell whether each sequence is arithmetic or geometric. Then find a formula for the *n*th term.

B
27. The sequence of positive even integers
28. The sequence of odd integers greater than two
29. 25, 33, 41, 49, . . .
30. −17, −11, −5, 1, . . .
31. 200, −100, 50, −25
32. $e^x, e^{2x}, e^{3x}, e^{4x}, \ldots$
33. $2a + 1, 3a + 3, 4a + 5, \ldots$
34. $\frac{a^2}{2}, \frac{a^4}{4}, \frac{a^6}{8}, \ldots$
35. The sequence of negative even integers that are multiples of 5
36. The sequence of positive integers that give a remainder of 1 when divided by 4

Find a formula for the *n*th term of each sequence. The sequences are neither arithmetic nor geometric. (*Hint:* Analyze the patterns in the numerators and denominators separately.)

37. $\frac{2}{1}, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \ldots$

38. $\frac{1}{2}, \frac{2}{5}, \frac{3}{8}, \frac{4}{11}, \ldots$

C **39.** Show that the sequence a, $\sqrt{ab}$, b (a, b positive) is geometric.

40. The first three numbers of the sequence 8, x, y, 36 form an arithmetic sequence, but the last three numbers form a geometric sequence. Find all possible values of x and y.

Problems

The following problems involve arithmetic and geometric sequences. A calculator may be helpful in solving these problems.

A **1.** Allysa has taken a job with a starting salary of $17,600 and annual raises of $850. What will be her salary during her fifth year on the job?

2. Frank has taken a job with a starting salary of $15,000 and annual raises of 4%. What will be his salary during his third year on the job?

3. The cost of an annual subscription to a magazine is $20 this year. The cost is expected to rise by 10% each year. What will be the cost 6 years from now?

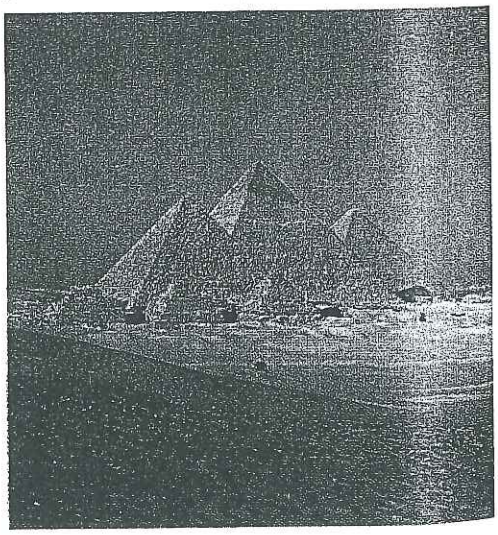
4. A new pair of running shoes costs $70 now. Assuming an annual 8% price increase, find the price 4 years from now.

5. A carpenter is building a staircase from the first floor to the second floor of a house. The distance between floors is 3.3 m. Each step rises 22 cm. Not counting either floor itself, how many steps will there be?

6. The width of a pyramid decreases by 1.57 m for each successive 1 m of height. If the width at a height of 1 m is 229.22 m, what is the width at a height of 86 m?

7. A culture of yeast doubles in size every 4 hours. If the yeast population is estimated to be 3 million now, what will it be one day from now?

8. An advertisement for a mutual fund claims that people who invested in the fund 5 years ago have doubled their money. If the fund's future performance is similar to its past performance, how much would a $2000 investment be worth in 40 years?



B 9. A pile of bricks has 85 bricks in the bottom row, 79 bricks in the second row, 73 in the third row, and so on until there is only 1 brick in the top row.

a. How many bricks are in the 12th row?

b. How many rows are there in all?

10. A projectile fired vertically upward rises 15,840 ft the first second, 15,808 ft the following second, and 15,776 ft the third second.

a. How many feet does it rise the 45th second?

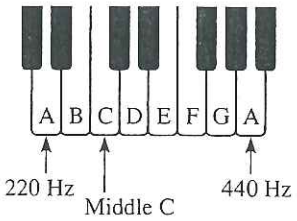
b. How many feet and in what direction does it move during the first second of the tenth minute after it has been fired?

11. A new \$12,000 automobile decreases in value by 25% each year. What is its value 7 years from now?

12. A house purchased last year for \$80,000 is now worth \$96,000. Assuming that the value of the house continues to appreciate (increase) at the same rate each year, find the value 2 years from now.

13. Job A has a starting salary of \$12,000 with annual increases of \$800. Job B has a starting salary of \$11,000 with annual increases of 10%. Which job will pay more after 3 years? after 5 years?

C 14. There are 12 steps in the chromatic scale from the A below middle C to the next higher A. This scale can be played by playing the white and black keys of a piano in order from left to right. The frequency of a note is given in hertz (Hz). For example, the A below middle C has a frequency of 220 Hz, which means the piano string vibrates 220 times per second. Notice that the frequency of the A above middle C is twice as great, 440 Hz. If the frequencies of the notes in the chromatic scale form a geometric sequence, show that the common ratio is $2^{1/12}$ and then find the frequency of middle C.



Mixed Review Exercises

Solve each inequality and graph the solution set.

1. $|2x - 5| > 9$
2. $x^2 + x \leq 6$
3. $2 \leq \frac{x}{2} + 4 < 5$
4. $x^3 > 4x$
5. $5 - 3x \geq -1$
6. $|x + 3| < 2$

Find an equation for each figure described.

7. The line containing $(-2, 3)$ and $(1, 0)$
8. The parabola with focus $(0, -1)$ and directrix $y = 1$
9. The ellipse with x-intercepts ± 2 and y-intercepts $\pm\sqrt{3}$

Series

11-4 Series and Sigma Notation

Objective To identify series and to use sigma notation.

When the terms of a sequence are added together, the resulting expression is called a **series.** Here are some examples:

Finite sequence	3, 7, 11, 15, 19
Related finite series	$3 + 7 + 11 + 15 + 19$
Infinite sequence	$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$
Related infinite series	$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$

Many of the words used to describe sequences are also used to describe series. For example, the finite series above is an arithmetic series with first term $t_1 = 3$, last term $t_5 = 19$, and general term $t_n = 4n - 1$. An **arithmetic series** is a series whose related sequence is arithmetic. Similarly, the infinite series above is a **geometric series** because its related sequence is geometric.

A series can be written in an abbreviated form by using the Greek letter Σ *(sigma),* called the **summation sign.** For instance, to abbreviate the writing of the series

$$2 + 4 + 6 + 8 + \cdots + 100$$

first notice that the general term of the series is $2n$. The series begins with the term for $n = 1$ and ends with the term for $n = 50$. Using sigma notation you can write this series as $\sum_{n=1}^{50} 2n$, which is read "the sum of $2n$ for values of n from 1 to 50."

$$\sum_{n=1}^{50} 2n = 2 \cdot 1 + 2 \cdot 2 + 2 \cdot 3 + 2 \cdot 4 + \cdots + 2 \cdot 50$$

$$= 2 + 4 + 6 + 8 + \cdots + 100$$

Similarly, the sigma expression below represents "the sum of n^2 for values of n from 1 to 10."

$$\sum_{n=1}^{10} n^2 = 1^2 + 2^2 + 3^2 + \cdots + 10^2$$

In sigma notation the general term, n^2, is called the **summand,** and the letter n is called the **index.**

Any letter can be used as the index. For example, replacing the index n by k in the series just given does not change the series:

$$\sum_{k=1}^{10} k^2 = 1^2 + 2^2 + 3^2 + \cdots + 10^2$$

Example 1 Write the series $\sum_{j=1}^{20} (-1)^j(j + 2)$ in expanded form.

Solution

$$\sum_{j=1}^{20} (-1)^j(j + 2) = (-1)^1(1 + 2) + (-1)^2(2 + 2) + (-1)^3(3 + 2) + \cdots + (-1)^{20}(20 + 2)$$

$$= -3 + 4 - 5 + \cdots + 22 \quad \textit{Answer}$$

The first and last values of the index are called the **limits of summation.** In Example 1, the *lower limit* is 1 and the *upper limit* is 20. If a series is infinite, then the symbol ∞ is used for the upper limit to indicate that the summation does not end. For example,

$$\sum_{k=1}^{\infty} \frac{1}{2^{k-1}}$$

is read "the sum of $\frac{1}{2^{k-1}}$ for values of k from 1 to infinity."

$$\sum_{k=1}^{\infty} \frac{1}{2^{k-1}} = \frac{1}{2^{1-1}} + \frac{1}{2^{2-1}} + \frac{1}{2^{3-1}} + \frac{1}{2^{4-1}} + \cdots$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

By changing the lower limit of summation from 1 to 0, this infinite geometric series can be rewritten with a simpler summand.

$$\sum_{k=0}^{\infty} \frac{1}{2^k} = \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

As you will see in Lesson 11-6, this infinite series has a finite sum of 2.

Example 2 Use sigma notation to write the series $10 + 15 + 20 + \cdots + 100$.

Solution 1 By inspection,

$$10 + 15 + 20 + \cdots + 100 = 5 \cdot 2 + 5 \cdot 3 + 5 \cdot 4 + \cdots + 5 \cdot 20.$$

$\therefore$ the series is $\sum_{k=2}^{20} 5k$. *Answer*

Solution 2 Since the series is arithmetic with common difference 5, the nth term is:

$$\begin{aligned} t_n &= 10 + (n - 1)5 \\ &= 5n + 5, \text{ or } 5(n + 1) \end{aligned}$$

Now find n such that the last term is 100.

$$\begin{aligned} t_n &= 5n + 5 \\ 100 &= 5n + 5 \\ n &= 19 \end{aligned}$$

$\therefore$ the series is $\sum_{n=1}^{19} 5(n + 1)$. *Answer*

In the two solutions of Example 2, notice that the expressions $\sum_{k=2}^{20} 5k$ and $\sum_{n=1}^{19} 5(n + 1)$ represent the same series.

Example 3 Use sigma notation to write the series $\frac{5}{2} - \frac{5}{4} + \frac{5}{6} - \frac{5}{8} + \cdots$.

Solution Since the infinite series is neither arithmetic nor geometric, you need to look for patterns:

(1) The numerators are all equal to 5.

(2) Since the denominators are consecutive even integers, a general expression for the denominators is $2n$.

(3) To make the terms of a series alternate in sign, choose $n = 1$ for the lower limit of the summand and include in the expression for the summand one of the following factors:

$(-1)^n$ makes *odd-numbered* terms *negative;*
$(-1)^{n+1}$ makes *even-numbered* terms *negative*.

So the expression for the summand is $(-1)^{n+1}\left(\frac{5}{2n}\right)$.

$\therefore$ the series is $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{5}{2n}\right)$. *Answer*

Oral Exercises

Exercises 1–4 refer to the series $\sum_{m=2}^{5} (4m^2 - 3)$.

1. What is the index?

2. What is the summand?

3. What are the limits of summation?

4. What are the first and last terms?

For each series, read each symbol aloud and give the expanded form.

5. $\sum_{i=1}^{3} i$

6. $\sum_{j=2}^{4} (5 - j)$

7. $\sum_{n=1}^{5} 4n$

8. $\sum_{k=1}^{4} k^3$

9. $\sum_{z=5}^{\infty} \frac{1}{z-1}$

10. $\sum_{j=0}^{6} (-1)^j$

Tell whether or not each pair of series is the same.

11. $\sum_{k=0}^{8} \frac{1}{k+3}, \sum_{j=3}^{11} \frac{1}{j}$

12. $\sum_{n=0}^{4} (-1)^n(n+1)^2, \sum_{j=1}^{5} (-1)^j(j^2)$

13. $\sum_{k=0}^{4} \frac{3k+3}{k+6}, \sum_{i=1}^{3} \frac{3i}{i+5}$

14. $\sum_{k=0}^{\infty} \frac{k+1}{3^k}, \sum_{n=1}^{\infty} \frac{n}{3^{n-1}}$

Written Exercises

Write each series in expanded form.

A 1. $\sum_{n=1}^{6} (n + 10)$

2. $\sum_{k=1}^{8} 3k$

3. $\sum_{n=1}^{6} 2^n$

4. $\sum_{n=4}^{10} (3n - 2)$

5. $\sum_{j=0}^{5} \frac{(-1)^j}{j+1}$

6. $\sum_{k=0}^{3} 4^{-k}$

7. $\sum_{n=3}^{8} |5 - n|$

8. $\sum_{k=1}^{4} (-k)^{k+1}$

Write each series using sigma notation.

9. $2 + 4 + 6 + \cdots + 1000$

10. $5 + 10 + 15 + \cdots + 250$

11. $1^3 + 2^3 + 3^3 + \cdots + 20^3$

12. $3 \cdot 4 + 3 \cdot 4^2 + 3 \cdot 4^3 + \cdots + 3 \cdot 4^{24}$

13. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots + \frac{99}{100}$

14. $\frac{1}{5^3} + \frac{1}{5^4} + \frac{1}{5^5} + \cdots + \frac{1}{5^{15}}$

15. $1 + 3 + 5 + \cdots + 199$

16. $3 + 7 + 11 + 15 + \cdots + 399$

17. $1 + 2 + 4 + 8 + \cdots + 64$

18. $\sqrt{7} + \sqrt{14} + \sqrt{21} + \cdots + \sqrt{77}$

19. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

20. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$

B 21. $-9 + 3 - 1 + \frac{1}{3} + \cdots$

22. $8 - 4 + 2 - 1 + \cdots$

23. $6 - 12 + 24 - 48$

24. $2 + 5 + 10 + 17 + 26 + 37$

25. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$

26. $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$

27. The series consisting of positive three-digit integers divisible by 5

28. The series consisting of positive two-digit integers ending in 2

17. a. $\frac{\log 56}{\log 5.6}$ **b.** 2.34 **19. a.** $-\frac{\log 5}{\log 30}$ **b.** -0.473 **21. a.** $\frac{\log 60}{2 \log 3.5}$ **b.** 1.63 **23.** $\left\{\frac{7}{4}\right\}$ **25.** $\left\{\frac{5}{6}\right\}$ **27.** {6740} **29.** {21.6} **31.** {2.19} **33.** {1.03} **35.** 3.17 **37.** 3.36 **39.** {0.631, 1.46} **41. a.** 2; $\frac{1}{2}$ **b.** 3; $\frac{1}{3}$ **c.** They are reciprocals. $\log_b a = \frac{\log_a a}{\log_a b} = \frac{1}{\log_a b}$

Problems, pages 486–488 **1. a.** \$1120 **b.** \$1254.40 **c.** \$1404.93 **d.** \$3105.85 **3. a.** \$1125.51 **b.** \$1266.77 **c.** \$1425.76 **d.** \$3262.04 **5. a.** \$10,000 **b.** \$8000 **c.** \$6400 **d.** \$1342.18 **7. a.** $4N_0$ **b.** $32N_0$ **c.** $(2^{w/3})N_0$ **9. a.** 25 kg **b.** 6.25 kg **c.** $100\left(\frac{1}{2}\right)^{y/6000}$ kg **11.** about 9 years **13.** 12.6% **15.** 0.997; 9.09×10^{-13} **17.** 11.4% **19.** 19.7% **21. a.** \$1,065,552.45 **b.** \$1,066,086.39 **c.** \$1,066,091.81

Mixed Review Exercises, page 488
1. $\left\{0, \frac{1}{2}, 1\right\}$ **2.** {1, 2} **3.** $\{\sqrt{2}, 2, 4\}$ **4.** {0, 6} **5.** $\left\{\frac{1}{3}, \frac{1}{2}, \frac{2}{3}\right\}$ **6.** $\{1, \sqrt{2}, 2\}$ **7.** 1 **8.** 2 **9.** $\frac{1}{6}$ **10.** 1 **11.** $\frac{5}{12}$ **12.** 3

Written Exercises, pages 490–491 **1.** $e^{2.08} = 8$ **3.** $e^{-1.10} = \frac{1}{3}$ **5.** $\ln 20.1 = 3$ **7.** $\ln 1.65 = \frac{1}{2}$ **9.** 2 **11.** -3 **13.** 0 **15.** 5 **17.** ln 12 **19.** $\ln \frac{9}{5}$ **21.** $\ln 10e^3$ **23.** $\{e^3\}$ **25.** $\{4 + e^{-1}\}$ **27.** $\{\pm e^{9/2}\}$ **29.** {ln 2} **31.** {ln 5} **33.** {2 + ln 2} **35.** {ln 9} **37.** $\left\{\frac{3}{5} \ln 10\right\}$ **39.** $\{e\}$ **41.** {2} **43.** {ln 3, ln 4} **45.** $D = \{x: x \neq 0\}$; $R = \{\text{reals}\}$ **47.** $D = \{x: x > 5\}$; $R = \{\text{reals}\}$

Calculator Key-In, page 492 **1. a.** \$1061.84 **b.** \$1127.50 **c.** \$1822.12 **3. a.** 1; 0.99; 0.96; 0.91; 0.85; 0.78; 0.70; 0.61; 0.53; 0.44; 0.37 **5.** 2.718056; e

Self-Test 3, page 493 **1.** {650} **2.** {5.81} **3.** {1.07} **4.** \$3277.23 **5.** $\ln\left(\frac{5}{e^2}\right)^{1/3}$ $\left(\text{or } \frac{\ln 5 - 2}{3}\right)$

Application, pages 493–494 **1.** 6600 **3. a.** 95.3% **b.** 93.0%

Chapter Review, pages 496–497 **1.** c **3.** d **5.** b **7.** a **9.** b **11.** a **13.** d **15.** b

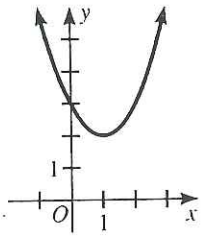
Mixed Problem Solving, page 498 **1.** 10 mL **3.** CA:45; NY:34; NC:11 **5.** -9 **7.** \$1500 **9.** $A = \frac{C^2}{4\pi}$ **11.** Larger plant: 8.4 h; smaller: 12.4 h **13.** 4 h

Preparing for College Entrance Exams, page 499 **1.** C **3.** E **5.** A **7.** E

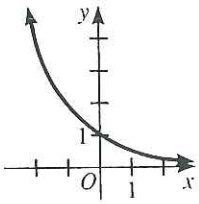
Chapter 11 Sequences and Series

Written Exercises, pages 504–506 **1.** A; 8, 5 **3.** G; 625, 3125 **5.** A; 30, 38 **7.** N; $\frac{1}{25}, \frac{1}{36}$ **9.** G; $4^{9/2}, 4^{11/2}$ **11.** 7, 11, 15, 19; A **13.** 1, 3, 9, 27; G **15.** $-\frac{1}{4}, \frac{1}{2}, -1, 2$; G **17.** log 2, log 3, log 4, log 5; N **19. a.** A **b.** G **21.** 32, 44 **23.** 20, 18 **25.** 63; 127 **27.** 21, 34 **29.** 48, 71 **31. a.** 15, 21 **b.** 55 **33. a.** 9 **b.** 35 **35. a.** 16, 31; No **b.** 57

Mixed Review Exercises, page 506
1.



2.



7. (2, -3) **8.** $\left(-\frac{3}{2}, \frac{7}{2}\right)$, (1, 1) **9.** (2, 1), (-2, 1), (2, -1), (-2, -1)

Written Exercises, page 509 **1.** $t_n = 8n + 16$ **3.** $t_n = 4 - 7n$ **5.** $t_n = 4n + 3$ **7.** 104 **9.** 52 **11.** -902 **13.** 87.5 **15.** 8 **17.** -61 **19.** 2 **21.** $\frac{3}{2}$ **23. a.** -7, 13 **b.** -12, 3, 18 **c.** -15, -3, 9, 21 **25. a.** 19, 27 **b.** 17, 23, 29 **c.** 15.8, 20.6, 25.4, 30.2 **27.** 101 **29.** 300 **31.** 16 **33.** $\frac{a+b}{2} - a = b - \frac{a+b}{2} = \frac{b-a}{2}$

Written Exercises, pages 513–514
1. $t_n = 2 \cdot 3^{n-1}$ **3.** $t_n = (\sqrt{2})^{n-1}$
5. $t_n = 64\left(-\frac{3}{4}\right)^{n-1}$ **7.** 39,366 **9.** $\frac{5}{256}$
11. $\frac{5}{128}$ **13.** $\frac{16}{3}$ **15.** 5120 **17.** y^{39} **19.** 4
21. 3 **23.** 10; 20; 40 **25.** $\sqrt[5]{2}$, $\sqrt[5]{4}$, $\sqrt[5]{8}$, $\sqrt[5]{16}$ **27.** A; $t_n = 2n$ **29.** A;
$t_n = 25 + 8(n - 1)$ **31.** G; $t_n = 200\left(-\frac{1}{2}\right)^{n-1}$
33. A; $t_n = (n + 1)a + (2n - 1)$ **35.** A;
$t_n = -10n$ **37.** $t_n = \frac{n+1}{n^2}$
39. $\frac{\sqrt{ab}}{a} = \frac{b}{\sqrt{ab}} = \sqrt{\frac{b}{a}}$

Problems, pages 514–515 **1.** \$21,000
3. \$35.43 **5.** 14 **7.** 192 million **9. a.** 19
b. 15 **11.** \$1601.87 **13.** B; B

Mixed Review Exercises, page 515
1. $\{x: x < -2 \text{ or } x > 7\}$ **2.** $\{x: -3 \le x \le 2\}$
3. $\{x: -4 \le x < 2\}$
4. $\{x: -2 < x < 0 \text{ or } x > 2\}$ **5.** $\{x: x \le 2\}$
6. $\{x: -5 < x < -1\}$ **7.** $x + y = 1$
8. $y = -\frac{1}{4}x^2$ **9.** $\frac{x^2}{4} + \frac{y^2}{3} = 1$

Self-Test 1, page 516 **1. a.** G; $\frac{27}{8}$, $\frac{81}{16}$ **b.** A;
-11, -15 **c.** N; $\frac{5}{11}$, $\frac{6}{13}$ **d.** A; 2.2, 2.5
2. a. $t_n = 43 - 7n$ **b.** -90 **3. a.** 4 **b.** 2, 6
4. a. $t_n = 48\left(-\frac{1}{2}\right)^{n-1}$ **b.** $-\frac{3}{32}$ **5. a.** $4\sqrt{2}$
b. 4, 8 **6.** \$58.32

Extra, page 517 **1.** 6 **3.** -2
5. {real numbers}; {positive integers}
7. {real numbers}; {positive integers}

Written Exercises, pages 521–522
1. $11 + 12 + 13 + 14 + 15 + 16$
3. $2 + 4 + 8 + 16 + 32 + 64$
5. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}$
7. $2 + 1 + 0 + 1 + 2 + 3$ **9.** $\sum_{n=1}^{500} 2n$
11. $\sum_{n=1}^{20} n^3$ **13.** $\sum_{n=1}^{99} \frac{n}{n+1}$ **15.** $\sum_{n=1}^{100} (2n - 1)$
17. $\sum_{n=0}^{6} 2^n$ **19.** $\sum_{n=1}^{\infty} \frac{1}{n}$ **21.** $\sum_{n=1}^{\infty} 27\left(-\frac{1}{3}\right)^n$
23. $\sum_{n=0}^{3} 6(-2)^n$ **25.** $\sum_{n=1}^{\infty} \frac{1}{n^2}$ **27.** $\sum_{n=20}^{199} 5n$
29. $\sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$ **31.** $\frac{j+4}{j+8}$ **33.** $\sum_{k=1}^{4} k \log 5 =$
$\log 5 + 2 \log 5 + 3 \log 5 + 4 \log 5 =$
$10 \log 5 = \log 5^{10}$ **35.** 21

Written Exercises, pages 527–528 **1.** 670
3. 3420 **5.** 25,250 **7.** 3925 **9.** 15,250
11. 893 **13.** 255 **15.** $-29{,}524$ **17.** $\frac{4095}{4096}$
19. a. -300 **b.** $\frac{3069}{64}$ **21.** 1960 **23.** 247,500
25. a. 420 **b.** 2,097,150 **27.** 3240
29. a. $\sum_{k=1}^{n} 2^{k-1} = \frac{2^0(1 - 2^n)}{1 - 2} = \frac{1 - 2^n}{-1} = 2^n - 1$
b. 20 **31.** 16, 48, 144; geometric
33. Answers will vary. **a.** 0.00000095
b. 0.00002656 **c.** 0.00001427 **35.** -50
37. 2,097,360

Problems, pages 528–530 **1.** 880 **3.** 156
5. 2046 **7.** Plan B; \$58.83 more **9.** B; A
11. \$13,971.64 **13. a.** $T_n = \frac{n(n+1)}{2}$
b. $T_n = \sum_{k=1}^{n} k$ **15.** the square of an odd number

Mixed Review Exercises, page 530 **1.** $\frac{\sqrt{2}}{4}$
2. 0 **3.** -2 **4.** 4 **5.** -1 **6.** $2\sqrt{17}$ **7.** 16
8. -1 **9.** -2 **10.** $\frac{3}{2}$ **11.** 4 **12.** (3, 0) and
$(-3, 0)$

Computer Exercises, page 530 **2. a.** 506
b. 1.99998957 **c.** 0.740049503

Written Exercises, pages 533–534 **1.** 48
3. 16.2 **5.** no sum **7.** no sum
9. $\frac{9}{2}(\sqrt{3} - 1)$ **11.** 4 **13.** 8, 10, $\frac{21}{2}$, $\frac{85}{8}$, $\frac{341}{32}$;
$S \approx 11$; $S = \frac{32}{3}$ **15.** $\frac{3}{4}$, $\frac{3}{8}$, $\frac{9}{16}$, $\frac{15}{32}$, $\frac{33}{64}$; $S \approx \frac{1}{2}$;
$S = \frac{1}{2}$ **17.** $\frac{1}{3}$ **19.** $\frac{1040}{333}$ **21.** $\frac{1}{2}$ **23.** 8, $\frac{8}{3}$, $\frac{8}{9}$
25. 40, $-\frac{40}{3}$, $\frac{40}{9}$ **29. a.** 3 **b.** 32

Problems, pages 534–536 **1.** 72 ft
3. $48(2 + \sqrt{2})$ cm **5.** 288 cm^2 **7.** 480 m
9. a. 0.18889 in. **b.** 249.24 in. **c.** 250 in.

35. 50 **37.** 21 **39.** 90 **41.** 26 **43.** 42 **45.** 96 **47.** $3 + 4 + \cdots + (n + 2)$ **49.** $\frac{1}{2} + 2 + \frac{9}{2} + \cdots + \frac{n^2}{2}$ **51.** $1 + \frac{1}{3} + \frac{1}{9} + \cdots + \frac{1}{3^n}$

53. $\frac{1}{3} + \frac{1}{9} + \cdots + \frac{1}{3^n}$ **55.** $\ln 2 - \ln 3 + \ln 4 - \cdots + (-1)^n \ln n$ **57.** $\sum_{k=1}^{20} k$ **59.** $\sum_{k=1}^{13} \frac{k}{k+1}$ **61.** $\sum_{k=0}^{6} (-1)^k \left(\frac{1}{3^k}\right)$ **63.** $\sum_{k=1}^{n} \frac{3^k}{k}$

65. $\sum_{k=0}^{n} (a + kd)$ or $\sum_{k=1}^{n+1} [a + (k-1)d]$ **67.** **(a)** \$2930 **(b)** 14 payments have been made. **(c)** 36 payments. \$3584.62 **(d)** \$584.62

69. **(a)** 2080 **(b)** After 26 months. **71.** \$529,244.62 **73.** **(a)** $a_0 = 0, a_n = (1.02)a_{n-1} + 500$ **(b)** After 82 quarters **(c)** \$156,116.15

75. **(a)** $a_0 = 150{,}000, a_n = (1.005)a_{n-1} - 899.33$
(b) \$149,850.67
(c)

n	u(n)
0	150000
1	149851
2	149701
3	149550
4	149398
5	149246
6	149093

n=0

(d) After 58 payments
(e) 30 years
(f) \$173,758.80

(g) **(a)** $a_0 = 150{,}000, a_n = (1.005)a_{n-1} - 999.33$
(b) \$149,750.67
(c)

n	u(n)
0	150000
1	149751
2	149500
3	149248
4	148995
5	148741
6	148485

n=0

(d) After 37 months
(e) After 279 payments, last payment: \$353.69(1.005) = \$355.46
(f) \$128,169.20

77. 21
79. A Fibonacci sequence

12.2 Exercises

1. $d = 1; 5, 6, 7, 8$ **3.** $d = 2; -3, -1, 1, 3$ **5.** $d = -2; 4, 2, 0, -2$ **7.** $d = -\frac{1}{3}; \frac{1}{6}, -\frac{1}{6}, -\frac{1}{2}, -\frac{5}{6}$ **9.** $d = \ln 3; \ln 3, 2\ln 3, 3\ln 3, 4\ln 3$

11. $a_n = 3n - 1; a_5 = 14$ **13.** $a_n = 8 - 3n; a_5 = -7$ **15.** $a_n = \frac{1}{2}(n-1); a_5 = 2$ **17.** $a_n = \sqrt{2}n; a_5 = 5\sqrt{2}$ **19.** $a_{12} = 24$

21. $a_{10} = -26$ **23.** $a_8 = a + 7b$ **25.** $a_1 = -13; d = 3; a_n = a_{n-1} + 3$ **27.** $a_1 = -53, d = 6; a_n = a_{n-1} + 6$

29. $a_1 = 28; d = -2; a_n = a_{n-1} - 2$ **31.** $a_1 = 25; d = -2; a_n = a_{n-1} - 2$ **33.** n^2 **35.** $\frac{n}{2}(9 + 5n)$ **37.** 1260 **39.** 324

41.
```
sum(seq(3.45n+4.
12,n,1,20,1))
              806.9
```

43.
```
sum(seq(2.4n+.4,
n,1,15,1))
                294
```

45.
```
sum(seq(2.58n+2.
32,n,1,25,1))
              896.5
```

47. $-\frac{3}{2}$
49. 1185 seats
51. 210 of beige and 190 blue
53. 30 rows

Historical Problems

1. $1\frac{2}{3}, 10\frac{5}{6}, 20, 29\frac{1}{6}, 38\frac{1}{3}$

12.3 Exercises

1. $r = 3; 3, 9, 27, 81$ **3.** $r = \frac{1}{2}; -\frac{3}{2}, -\frac{3}{4}, -\frac{3}{8}, -\frac{3}{16}$ **5.** $r = 2; \frac{1}{4}, \frac{1}{2}, 1, 2$ **7.** $r = 2^{1/3}; 2^{1/3}, 2^{2/3}, 2, 2^{4/3}$ **9.** $r = \frac{3}{2}; \frac{1}{2}, \frac{3}{4}, \frac{9}{8}, \frac{27}{16}$

11. Arithmetic; $d = 1$ **13.** Neither **15.** Arithmetic; $d = -\frac{2}{3}$ **17.** Neither **19.** Geometric; $r = \frac{2}{3}$ **21.** Geometric; $r = 2$

23. Geometric; $r = 3^{1/2}$ **25.** $a_5 = 162; a_n = 2 \cdot 3^{n-1}$ **27.** $a_5 = 5; a_n = 5 \cdot (-1)^{n-1}$ **29.** $a_5 = 0; a_n = 0$ **31.** $a_5 = 4\sqrt{2}; a_n = (\sqrt{2})^n$

33. $a_7 = \frac{1}{64}$ **35.** $a_9 = 1$ **37.** $a_8 = 0.00000004$ **39.** $-\frac{1}{4}(1 - 2^n)$ **41.** $2\left[1 - \left(\frac{2}{3}\right)^n\right]$ **43.** $1 - 2^n$

45.
```
(1/4)sum(seq(2^n
,n,0,14,1))
            8191.75
```

47.
```
sum(seq((2/3)^n,
n,1,15,1))
   1.995432683
```

49.
```
-1sum(seq(2^n,n,
0,14,1))
            -32767
```

51. $\frac{3}{2}$ **53.** 16 **55.** $\frac{8}{5}$
57. $\frac{20}{3}$ **59.** $\frac{18}{5}$ **61.** -4

63. 10 **65.** \$72.67 **67.** **(a)** 0.775 ft **(b)** 8th **(c)** 15.88 ft **(d)** 20 ft **69.** \$21,879
71. Option 2 results in the most: \$16,038,304; Option 1 results in the least: \$14,700,000 **73.** 1.845×10^{19} **77.** 3, 5, 9, 24, 73
79. *A*: \$25,250 per year in 5th year, \$112,742 total; *B*: \$24,761 per year in 5th year, \$116,801 total