

KEY

Rules for "finding" the nth term:

Fill in the following tables and answer the questions.

1. Linear (arithmetic) sequences

a.) $u_n = 3n + 2$

n	1	2	3	4	5
u_n	5	8	11	14	17
1 st difference	3	3	3	3	3
2 nd difference	0	0	0	0	0

b.) $u_n = -2n + 3$

n	1	2	3	4	5
u_n	1	-1	-3	-5	-7
1 st difference	-2	-2	-2	-2	-2
2 nd difference	0	0	0	0	0

c.) $u_n = an + b$

n	1	2	3	4	5
u_n	$a + b$	$2a + b$	$3a + b$	$4a + b$	$5a + b$
1 st difference	a	a	a	a	a
2 nd difference	0	0	0	0	0

^{typo} The FIRST difference in a linear sequence is always constant!

2. Quadratic sequences

(Remember when we dealt with quadratic functions, we used the general form

$$f(x) = ax^2 + bx + c$$

a.) $u_n = n^2 + 3n + 3$

n	1	2	3	4	5
u_n	7	13	21	31	43
1 st difference	6	8	10	12	
2 nd difference	2	2	2		

b.) $u_n = -2n^2 + 3$

n	1	2	3	4	5
u_n	1	-5	-15	-29	-47
1 st difference		-6	-10	-14	-18
2 nd difference			-4	-4	-4

c.) $u_n = an^2 + bn + c$

n	1	2	3	4	5
u_n	$a + b + c$	$4a + 2b + c$	$9a + 3b + c$	$16a + 4b + c$	$25a + 5b + c$
1 st difference		$3a + b$	$5a + b$	$7a + b$	$9a + b$
2 nd difference			$2a$	$2a$	$2a$

The SECOND difference for a quadratic sequence is always *constant*.

PREDICT what the THIRD difference will be for a CUBIC sequence.

The THIRD difference for a cubic sequence is always *constant!*

Test your prediction with the following sequence.

3. a.) $u_n = 2n^3 + 3n^2 - 2n + 5$

n	1	2	3	4	5
u_n	8	29	80	173	320
1 st difference		21	51	93	147
2 nd difference			30	42	54
3 rd difference				12	12

b.) $u_n = an^3 + bn^2 + cn + d$

n	1	2	3	4	5
u_n	$a + b + c + d$	$8a + 4b + 2c + d$	$27a + 9b + 3c + d$	$64a + 16b + 4c + d$	$125a + 25b + 5c + d$
1 st difference		$7a + 3b + c$	$19a + 5b + c$	$37a + 7b + c$	$61a + 9b + c$
2 nd difference			$12a + 2b$	$18a + 2b$	$24a + 2b$
3 rd difference				$6a$	$6a$

more spacing!