

Prior learning topics

As noted in the previous section on prior learning, it is expected that all students have extensive previous mathematical experiences, but these will vary. It is expected that mathematics HL students will be familiar with the following topics before they take the examinations, because questions assume knowledge of them. Teachers must therefore ensure that any topics listed here that are unknown to their students at the start of the course are included at an early stage. They should also take into account the existing mathematical knowledge of their students to design an appropriate course of study for mathematics HL. This table lists the knowledge, together with the syllabus content, that is essential to successful completion of the mathematics HL course.

Students must be familiar with SI (*Système International*) units of length, mass and time, and their derived units.

Topic	Content
Number	<p>Routine use of addition, subtraction, multiplication and division, using integers, decimals and fractions, including order of operations.</p> <p>Rational exponents.</p> <p>Simplification of expressions involving roots (surds or radicals), including rationalizing the denominator.</p> <p>Prime numbers and factors (divisors), including greatest common divisors and least common multiples.</p> <p>Simple applications of ratio, percentage and proportion, linked to similarity.</p> <p>Definition and elementary treatment of absolute value (modulus), a.</p> <p>Rounding, decimal approximations and significant figures, including appreciation of errors.</p> <p>Expression of numbers in standard form (scientific notation), that is, $a \times 10^k$, $1 \leq a < 10$, $k \in \mathbb{Z}$.</p>
Sets and numbers	<p>Concept and notation of sets, elements, universal (reference) set, empty (null) set, complement, subset, equality of sets, disjoint sets. Operations on sets: union and intersection. Commutative, associative and distributive properties. Venn diagrams.</p> <p>Number systems: natural numbers; integers, \mathbb{Z}; rationals, \mathbb{Q}, and irrationals; real numbers, \mathbb{R}.</p> <p>Intervals on the real number line using set notation and using inequalities. Expressing the solution set of a linear inequality on the number line and in set notation.</p> <p>Mappings of the elements of one set to another; sets of ordered pairs.</p>

Topic	Content
Algebra	<p>Manipulation of linear and quadratic expressions, including factorization, expansion, completing the square and use of the formula.</p> <p>Rearrangement, evaluation and combination of simple formulae. Examples from other subject areas, particularly the sciences, should be included.</p> <p>Linear functions, their graphs, gradients and y-intercepts.</p> <p>Addition and subtraction of simple algebraic fractions.</p> <p>The properties of order relations: $<$, \leq, $>$, \geq.</p> <p>Solution of linear equations and inequalities in one variable, including cases with rational coefficients.</p> <p>Solution of quadratic equations and inequalities, using factorization and completing the square.</p> <p>Solution of simultaneous linear equations in two variables.</p>
Trigonometry	<p>Angle measurement in degrees. Compass directions. Right-angle trigonometry. Simple applications for solving triangles.</p> <p>Pythagoras' theorem and its converse.</p>
Geometry	<p>Simple geometric transformations: translation, reflection, rotation, enlargement.</p> <p>Congruence and similarity, including the concept of scale factor of an enlargement.</p> <p>The circle, its centre and radius, area and circumference. The terms arc, sector, chord, tangent and segment.</p> <p>Perimeter and area of plane figures. Properties of triangles and quadrilaterals, including parallelograms, rhombuses, rectangles, squares, kites and trapeziums (trapezoids); compound shapes. Volumes of cuboids, pyramids, spheres, cylinders and cones.</p> <p>Classification of prisms and pyramids, including tetrahedra.</p>
Coordinate geometry	<p>Elementary geometry of the plane, including the concepts of dimension for point, line, plane and space. The equation of a line in the form $y = mx + c$. Parallel and perpendicular lines, including $m_1 = m_2$ and $m_1 m_2 = -1$.</p> <p>The Cartesian plane: ordered pairs (x, y), origin, axes. Mid-point of a line segment and distance between two points in the Cartesian plane.</p>
Statistics and probability	<p>Descriptive statistics: collection of raw data, display of data in pictorial and diagrammatic forms, including frequency histograms, cumulative frequency graphs.</p> <p>Obtaining simple statistics from discrete and continuous data, including mean, median, mode, quartiles, range, interquartile range and percentiles.</p> <p>Calculating probabilities of simple events.</p>

Syllabus content

Topic I—Core: Algebra

30 hours

The aim of this topic is to introduce students to some basic algebraic concepts and applications.

	Content	Further guidance	Links
1.1	<p>Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series.</p> <p>Sigma notation.</p> <p>Applications.</p>	<p>Sequences can be generated and displayed in several ways, including recursive functions.</p> <p>Link infinite geometric series with limits of convergence in 6.1.</p> <p>Examples include compound interest and population growth.</p>	<p>Int: The chess legend (Sissa ibn Dahir).</p> <p>Int: Aryabhata is sometimes considered the “father of algebra”. Compare with al-Khawarizmi.</p> <p>Int: The use of several alphabets in mathematical notation (eg first term and common difference of an arithmetic sequence).</p> <p>TOK: Mathematics and the knower. To what extent should mathematical knowledge be consistent with our intuition?</p> <p>TOK: Mathematics and the world. Some mathematical constants (π, e, ϕ, Fibonacci numbers) appear consistently in nature. What does this tell us about mathematical knowledge?</p> <p>TOK: Mathematics and the knower. How is mathematical intuition used as a basis for formal proof? (Gauss’ method for adding up integers from 1 to 100.)</p> <p style="text-align: right;"><i>(continued)</i></p>

	Content	Further guidance	Links
			<p>(see notes above)</p> <p>Aim 8: Short-term loans at high interest rates. How can knowledge of mathematics result in individuals being exploited or protected from extortion?</p> <p>Appl: Physics 7.2, 13.2 (radioactive decay and nuclear physics).</p>
1.2	<p>Exponents and logarithms.</p> <p>Laws of exponents; laws of logarithms.</p> <p>Change of base.</p>	<p>Exponents and logarithms are further developed in 2.4.</p>	<p>Appl: Chemistry 18.1, 18.2 (calculation of pH and buffer solutions).</p> <p>TOK: The nature of mathematics and science. Were logarithms an invention or discovery? (This topic is an opportunity for teachers and students to reflect on “the nature of mathematics”.)</p>
1.3	<p>Counting principles, including permutations and combinations.</p> <p>The binomial theorem: expansion of $(a + b)^n$, $n \in \mathbb{N}$.</p> <p>Not required: Permutations where some objects are identical. Circular arrangements. Proof of binomial theorem.</p>	<p>The ability to find $\binom{n}{r}$ and nP_r using both the formula and technology is expected. Link to 5.4.</p> <p>Link to 5.6, binomial distribution.</p>	<p>TOK: The nature of mathematics. The unforeseen links between Pascal’s triangle, counting methods and the coefficients of polynomials. Is there an underlying truth that can be found linking these?</p> <p>Int: The properties of Pascal’s triangle were known in a number of different cultures long before Pascal (eg the Chinese mathematician Yang Hui).</p> <p>Aim 8: How many different tickets are possible in a lottery? What does this tell us about the ethics of selling lottery tickets to those who do not understand the implications of these large numbers?</p>

	Content	Further guidance	Links
1.4	Proof by mathematical induction.	Links to a wide variety of topics, for example, complex numbers, differentiation, sums of series and divisibility.	<p>TOK: Nature of mathematics and science. What are the different meanings of induction in mathematics and science?</p> <p>TOK: Knowledge claims in mathematics. Do proofs provide us with completely certain knowledge?</p> <p>TOK: Knowledge communities. Who judges the validity of a proof?</p>
1.5	<p>Complex numbers: the number $i = \sqrt{-1}$; the terms real part, imaginary part, conjugate, modulus and argument.</p> <p>Cartesian form $z = a + ib$.</p> <p>Sums, products and quotients of complex numbers.</p>	When solving problems, students may need to use technology.	<p>Appl: Concepts in electrical engineering. Impedance as a combination of resistance and reactance; also apparent power as a combination of real and reactive powers. These combinations take the form $z = a + ib$.</p> <p>TOK: Mathematics and the knower. Do the words imaginary and complex make the concepts more difficult than if they had different names?</p> <p>TOK: The nature of mathematics. Has “i” been invented or was it discovered?</p> <p>TOK: Mathematics and the world. Why does “i” appear in so many fundamental laws of physics?</p>

	Content	Further guidance	Links
1.6	<p>Modulus–argument (polar) form $z = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta = r e^{i\theta}$.</p> <p>The complex plane.</p>	<p>$r e^{i\theta}$ is also known as Euler's form.</p> <p>The ability to convert between forms is expected.</p> <p>The complex plane is also known as the Argand diagram.</p>	<p>Appl: Concepts in electrical engineering. Phase angle/shift, power factor and apparent power as a complex quantity in polar form.</p> <p>TOK: The nature of mathematics. Was the complex plane already there before it was used to represent complex numbers geometrically?</p> <p>TOK: Mathematics and the knower. Why might it be said that $e^{i\pi} + 1 = 0$ is beautiful?</p>
1.7	<p>Powers of complex numbers: de Moivre's theorem.</p> <p>n^{th} roots of a complex number.</p>	Proof by mathematical induction for $n \in \mathbb{Z}^+$.	TOK: Reason and mathematics. What is mathematical reasoning and what role does proof play in this form of reasoning? Are there examples of proof that are not mathematical?
1.8	Conjugate roots of polynomial equations with real coefficients.	Link to 2.5 and 2.7.	
1.9	<p>Solutions of systems of linear equations (a maximum of three equations in three unknowns), including cases where there is a unique solution, an infinity of solutions or no solution.</p> <p>(+ matrices?) (extra)</p>	<p>These systems should be solved using both algebraic and technological methods, eg row reduction.</p> <p>Systems that have solution(s) may be referred to as consistent.</p> <p>When a system has an infinity of solutions, a general solution may be required.</p> <p>Link to vectors in 4.7.</p>	TOK: Mathematics, sense, perception and reason. If we can find solutions in higher dimensions, can we reason that these spaces exist beyond our sense perception?

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Topic 2—Core: Functions and equations

22 hours

The aims of this topic are to explore the notion of function as a unifying theme in mathematics, and to apply functional methods to a variety of mathematical situations. It is expected that extensive use will be made of technology in both the development and the application of this topic.

	Content	Further guidance	Links
2.1	<p>Concept of function $f : x \mapsto f(x)$: domain, range; image (value).</p> <p>Odd and even functions.</p> <p>Composite functions $f \circ g$.</p> <p>Identity function.</p> <p>One-to-one and many-to-one functions.</p> <p>Inverse function f^{-1}, including domain restriction. Self-inverse functions.</p>	<p>$(f \circ g)(x) = f(g(x))$. Link with 6.2.</p> <p>Link with 3.4.</p> <p>Link with 6.2.</p>	<p>Int: The notation for functions was developed by a number of different mathematicians in the 17th and 18th centuries. How did the notation we use today become internationally accepted?</p> <p>TOK: The nature of mathematics. Is mathematics simply the manipulation of symbols under a set of formal rules?</p>

	Content	Further guidance	Links
2.2	<p>The graph of a function; its equation $y = f(x)$.</p> <p>Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes and symmetry, and consideration of domain and range.</p> <p>The graphs of the functions $y = f(x)$ and $y = f(x)$.</p> <p>The graph of $y = \frac{1}{f(x)}$ given the graph of $y = f(x)$.</p>	Use of technology to graph a variety of functions.	<p>TOK: Mathematics and knowledge claims. Does studying the graph of a function contain the same level of mathematical rigour as studying the function algebraically (analytically)?</p> <p>Appl: Sketching and interpreting graphs; Geography SL/HL (geographic skills); Chemistry 11.3.1.</p> <p>Int: Bourbaki group analytical approach versus Mandelbrot visual approach.</p>
2.3	<p>Transformations of graphs: translations; stretches; reflections in the axes.</p> <p>The graph of the inverse function as a reflection in $y = x$.</p>	Link to 3.4. Students are expected to be aware of the effect of transformations on both the algebraic expression and the graph of a function.	Appl: Economics SL/HL 1.1 (shift in demand and supply curves).
2.4	<p>The rational function $x \mapsto \frac{ax+b}{cx+d}$, and its graph.</p> <p>The function $x \mapsto a^x$, $a > 0$, and its graph.</p> <p>The function $x \mapsto \log_a x$, $x > 0$, and its graph.</p>	<p>The reciprocal function is a particular case.</p> <p>Graphs should include both asymptotes and any intercepts with axes.</p> <p>Exponential and logarithmic functions as inverses of each other.</p> <p>Link to 6.2 and the significance of e.</p> <p>Application of concepts in 2.1, 2.2 and 2.3.</p>	<p>Appl: Geography SL/HL (geographic skills); Physics SL/HL 7.2 (radioactive decay); Chemistry SL/HL 16.3 (activation energy); Economics SL/HL 3.2 (exchange rates).</p>

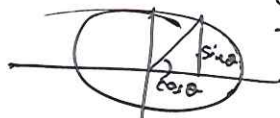
	Content	Further guidance	Links
2.5	<p>Polynomial functions and their graphs.</p> <p>The factor and remainder theorems.</p> <p>The fundamental theorem of algebra.</p>	<p>The graphical significance of repeated factors.</p> <p>The relationship between the degree of a polynomial function and the possible numbers of x-intercepts.</p>	
2.6	<p>Solving quadratic equations using the quadratic formula.</p> <p>Use of the discriminant $\Delta = b^2 - 4ac$ to determine the nature of the roots.</p> <p>Solving polynomial equations both graphically and algebraically.</p> <p>Sum and product of the roots of polynomial equations.</p> <p>Solution of $a^x = b$ using logarithms.</p> <p>Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach.</p>	<p>May be referred to as roots of equations or zeros of functions.</p> <p>Link the solution of polynomial equations to conjugate roots in 1.8.</p> <p>For the polynomial equation $\sum_{r=0}^n a_r x^r = 0$,</p> <p>the sum is $\frac{-a_{n-1}}{a_n}$,</p> <p>the product is $\frac{(-1)^n a_0}{a_n}$.</p> <p><i>(Vieta's formula)</i></p>	<p>Appl: Chemistry 17.2 (equilibrium law).</p> <p>Appl: Physics 2.1 (kinematics).</p> <p>Appl: Physics 4.2 (energy changes in simple harmonic motion).</p> <p>Appl: Physics (HL only) 9.1 (projectile motion).</p> <p>Aim 8: The phrase “exponential growth” is used popularly to describe a number of phenomena. Is this a misleading use of a mathematical term?</p>

	Content	Further guidance	Links
2.7	<p>Solutions of $g(x) \geq f(x)$.</p> <p>Graphical or algebraic methods, for simple polynomials up to degree 3.</p> <p>Use of technology for these and other functions.</p>		

Topic 3—Core: Circular functions and trigonometry

22 hours

The aims of this topic are to explore the circular functions, to introduce some important trigonometric identities and to solve triangles using trigonometry. On examination papers, radian measure should be assumed unless otherwise indicated, for example, by $x \mapsto \sin x^\circ$.

	Content	Further guidance	Links
3.1	The circle: radian measure of angles. Length of an arc; area of a sector.	Radian measure may be expressed as multiples of π , or decimals. Link with 6.2.	Int: The origin of degrees in the mathematics of Mesopotamia and why we use minutes and seconds for time.
3.2	Definition of $\cos \theta$, $\sin \theta$ and $\tan \theta$ in terms of the unit circle. Exact values of \sin , \cos and \tan of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples. Definition of the reciprocal trigonometric ratios $\sec \theta$, $\csc \theta$ and $\cot \theta$. Pythagorean identities: $\cos^2 \theta + \sin^2 \theta = 1$; $1 + \tan^2 \theta = \sec^2 \theta$; $1 + \cot^2 \theta = \csc^2 \theta$.	<i>Note: I have defined $\sin \theta$ & $\cos \theta$ & simply used that $\tan \theta = \frac{\sin \theta}{\cos \theta}$</i> 	TOK: Mathematics and the knower. Why do we use radians? (The arbitrary nature of degree measure versus radians as real numbers and the implications of using these two measures on the shape of sinusoidal graphs.) TOK: Mathematics and knowledge claims. If trigonometry is based on right triangles, how can we sensibly consider trigonometric ratios of angles greater than a right angle? Int: The origin of the word “sine”.
3.3	Compound angle identities. Double angle identities. Not required: Proof of compound angle identities.	Derivation of double angle identities from compound angle identities. Finding possible values of trigonometric ratios without finding θ , for example, finding $\sin 2\theta$ given $\sin \theta$.	Appl: Physics SL/HL 2.2 (forces and dynamics). Appl: Triangulation used in the Global Positioning System (GPS). Int: Why did Pythagoras link the study of music and mathematics? Appl: Concepts in electrical engineering. Generation of sinusoidal voltage.

(continued)

	Content	Further guidance	Links
3.4	Composite functions of the form $f(x) = a \sin(b(x+c)) + d$. Applications.		(see notes above) TOK: Mathematics and the world. Music can be expressed using mathematics. Does this mean that music is mathematical, that mathematics is musical or that both are reflections of a common “truth”? Appl: Physics SL/HL 4.1 (kinematics of simple harmonic motion).
3.5	The inverse functions $x \mapsto \arcsin x$, $x \mapsto \arccos x$, $x \mapsto \arctan x$; their domains and ranges; their graphs.		
3.6	Algebraic and graphical methods of solving trigonometric equations in a finite interval, including the use of trigonometric identities and factorization. Not required: The general solution of trigonometric equations.		TOK: Mathematics and knowledge claims. How can there be an infinite number of discrete solutions to an equation?
3.7	The cosine rule The sine rule including the ambiguous case. Area of a triangle as $\frac{1}{2}ab \sin C$. Applications.	Examples include navigation, problems in two and three dimensions, including angles of elevation and depression.	TOK: Nature of mathematics. If the angles of a triangle can add up to less than 180° , 180° or more than 180° , what does this tell us about the “fact” of the angle sum of a triangle and about the nature of mathematical knowledge? Appl: Physics SL/HL 1.3 (vectors and scalars); Physics SL/HL 2.2 (forces and dynamics). Int: The use of triangulation to find the curvature of the Earth in order to settle a dispute between England and France over Newton’s gravity.

Topic 4—Core: Vectors

24 hours

The aim of this topic is to introduce the use of vectors in two and three dimensions, and to facilitate solving problems involving points, lines and planes.

	Content	Further guidance	Links
4.1	<p>Concept of a vector.</p> <p>Representation of vectors using directed line segments.</p> <p>Unit vectors; base vectors i, j, k.</p> <p>Components of a vector:</p> $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}.$ <p>Algebraic and geometric approaches to the following:</p> <ul style="list-style-type: none"> the sum and difference of two vectors; the zero vector $\mathbf{0}$, the vector $-\mathbf{v}$; multiplication by a scalar, $k\mathbf{v}$; magnitude of a vector, \mathbf{v}; position vectors $\vec{OA} = \mathbf{a}$. <p>$\vec{AB} = \mathbf{b} - \mathbf{a}$</p>	<p>Proofs of geometrical properties using vectors.</p> <p>Distance between points A and B is the magnitude of \vec{AB}.</p>	<p>Aim 8: Vectors are used to solve many problems in position location. This can be used to save a lost sailor or destroy a building with a laser-guided bomb.</p> <p>Appl: Physics SL/HL 1.3 (vectors and scalars); Physics SL/HL 2.2 (forces and dynamics).</p> <p>TOK: Mathematics and knowledge claims. You can perform some proofs using different mathematical concepts. What does this tell us about mathematical knowledge?</p>

	Content	Further guidance	Links
4.2	<p>The definition of the scalar product of two vectors.</p> <p>Properties of the scalar product:</p> $\mathbf{v} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v};$ $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w};$ $(k\mathbf{v}) \cdot \mathbf{w} = k(\mathbf{v} \cdot \mathbf{w});$ $\mathbf{v} \cdot \mathbf{v} = \mathbf{v} ^2.$ <p>The angle between two vectors.</p> <p>Perpendicular vectors; parallel vectors.</p>	<p>$\mathbf{v} \cdot \mathbf{w} = \mathbf{v} \mathbf{w} \cos\theta$, where θ is the angle between \mathbf{v} and \mathbf{w}.</p> <p>Link to 3.6.</p> <p>For non-zero vectors, $\mathbf{v} \cdot \mathbf{w} = 0$ is equivalent to the vectors being perpendicular.</p> <p>For parallel vectors, $\mathbf{v} \cdot \mathbf{w} = \mathbf{v} \mathbf{w}$.</p>	<p>Appl: Physics SL/HL 2.2 (forces and dynamics).</p> <p>TOK: The nature of mathematics. Why this definition of scalar product?</p>
4.3	<p>Vector equation of a line in two and three dimensions: $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$.</p> <p>Simple applications to kinematics.</p> <p>The angle between two lines.</p>	<p>Knowledge of the following forms for equations of lines.</p> <p>Parametric form:</p> $x = x_0 + \lambda l, \quad y = y_0 + \lambda m, \quad z = z_0 + \lambda n.$ <p>Cartesian form:</p> $\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}.$	<p>Appl: Modelling linear motion in three dimensions.</p> <p>Appl: Navigational devices, eg GPS.</p> <p>TOK: The nature of mathematics. Why might it be argued that vector representation of lines is superior to Cartesian?</p>
4.4	<p>Coincident, parallel, intersecting and skew lines; distinguishing between these cases.</p> <p>Points of intersection.</p>		

More (including vector revision sheet!)

	Content	Further guidance	Links
4.5	<p>The definition of the vector product of two vectors.</p> <p>Properties of the vector product:</p> $\mathbf{v} \times \mathbf{w} = -\mathbf{w} \times \mathbf{v};$ $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w};$ $(k\mathbf{v}) \times \mathbf{w} = k(\mathbf{v} \times \mathbf{w});$ $\mathbf{v} \times \mathbf{v} = \mathbf{0}.$ <p>Geometric interpretation of $\mathbf{v} \times \mathbf{w}$.</p>	<p>$\mathbf{v} \times \mathbf{w} = \mathbf{v} \mathbf{w} \sin\theta \mathbf{n}$, where θ is the angle between \mathbf{v} and \mathbf{w} and \mathbf{n} is the unit normal vector whose direction is given by the right-hand screw rule.</p> <p>Areas of triangles and parallelograms.</p>	<p>Appl: Physics SL/HL 6.3 (magnetic force and field).</p>
4.6	<p>Vector equation of a plane $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$.</p> <p>Use of normal vector to obtain the form $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$. <i>more practice w/ this form.</i></p> <p>Cartesian equation of a plane $ax + by + cz = d$.</p>		
4.7	<p>Intersections of: a line with a plane; two planes; three planes.</p> <p>Angle between: a line and a plane; two planes.</p> <p><i>matrix algebra added here.</i></p>	<p>Link to 1.9.</p> <p>Geometrical interpretation of solutions.</p>	<p>TOK: Mathematics and the knower. Why are symbolic representations of three-dimensional objects easier to deal with than visual representations? What does this tell us about our knowledge of mathematics in other dimensions?</p>

Topic 5—Core: Statistics and probability

36 hours

The aim of this topic is to introduce basic concepts. It may be considered as three parts: manipulation and presentation of statistical data (5.1), the laws of probability (5.2–5.4), and random variables and their probability distributions (5.5–5.7). It is expected that most of the calculations required will be done on a GDC. The emphasis is on understanding and interpreting the results obtained. Statistical tables will no longer be allowed in examinations.

	Content	Further guidance	Links
5.1	<p>Concepts of population, sample, random sample and frequency distribution of discrete and continuous data.</p> <p>Grouped data: mid-interval values, interval width, upper and lower interval boundaries.</p> <p>Mean, variance, standard deviation.</p> <p>Not required: Estimation of mean and variance of a population from a sample.</p>	<p>For examination purposes, in papers 1 and 2 data will be treated as the population.</p> <p>In examinations the following formulae should be used:</p> $\mu = \frac{\sum_{i=1}^k f_i x_i}{n},$ $\sigma^2 = \frac{\sum_{i=1}^k f_i (x_i - \mu)^2}{n} = \frac{\sum_{i=1}^k f_i x_i^2}{n} - \mu^2.$	<p>TOK: The nature of mathematics. Why have mathematics and statistics sometimes been treated as separate subjects?</p> <p>TOK: The nature of knowing. Is there a difference between information and data?</p> <p>Aim 8: Does the use of statistics lead to an overemphasis on attributes that can easily be measured over those that cannot?</p> <p>Appl: Psychology SL/HL (descriptive statistics); Geography SL/HL (geographic skills); Biology SL/HL 1.1.2 (statistical analysis).</p> <p>Appl: Methods of collecting data in real life (census versus sampling).</p> <p>Appl: Misleading statistics in media reports.</p>

	Content	Further guidance	Links
5.2	<p>Concepts of trial, outcome, equally likely outcomes, sample space (U) and event.</p> <p>The probability of an event A as $P(A) = \frac{n(A)}{n(U)}$.</p> <p>The complementary events A and A' (not A).</p> <p>Use of Venn diagrams, tree diagrams, counting principles and tables of outcomes to solve problems.</p>		<p>Aim 8: Why has it been argued that theories based on the calculable probabilities found in casinos are pernicious when applied to everyday life (eg economics)?</p> <p>Int: The development of the mathematical theory of probability in 17th century France.</p>
5.3	<p>Combined events; the formula for $P(A \cup B)$.</p> <p>Mutually exclusive events.</p>		
5.4	<p>Conditional probability; the definition</p> $P(A B) = \frac{P(A \cap B)}{P(B)}.$ <p>Independent events; the definition</p> $P(A B) = P(A) = P(A B').$ <p>Use of Bayes' theorem for a maximum of three events.</p>	<p>Use of $P(A \cap B) = P(A)P(B)$ to show independence.</p>	<p>Appl: Use of probability methods in medical studies to assess risk factors for certain diseases.</p> <p>TOK: Mathematics and knowledge claims. Is independence as defined in probabilistic terms the same as that found in normal experience?</p>

	Content	Further guidance	Links
5.5	<p>Concept of discrete and continuous random variables and their probability distributions.</p> <p>Definition and use of probability density functions.</p> <p>Expected value (mean), mode, median, variance and standard deviation.</p> <p>Applications.</p>	<p>For a continuous random variable, a value at which the probability density function has a maximum value is called a mode.</p> <p>Examples include games of chance.</p>	<p>TOK: Mathematics and the knower. To what extent can we trust samples of data?</p> <p>Appl: Expected gain to insurance companies.</p>
5.6	<p>Binomial distribution, its mean and variance.</p> <p>Poisson distribution, its mean and variance.</p> <p>Not required: Formal proof of means and variances.</p>	<p>Link to binomial theorem in 1.3.</p> <p>Conditions under which random variables have these distributions.</p>	<p>TOK: Mathematics and the real world. Is the binomial distribution ever a useful model for an actual real-world situation?</p>
5.7	<p>Normal distribution.</p> <p>Properties of the normal distribution.</p> <p>Standardization of normal variables.</p>	<p>Probabilities and values of the variable must be found using technology.</p> <p>The standardized value (z) gives the number of standard deviations from the mean.</p> <p>Link to 2.3.</p>	<p>Appl: Chemistry SL/HL 6.2 (collision theory); Psychology HL (descriptive statistics); Biology SL/HL 1.1.3 (statistical analysis).</p> <p>Aim 8: Why might the misuse of the normal distribution lead to dangerous inferences and conclusions?</p> <p>TOK: Mathematics and knowledge claims. To what extent can we trust mathematical models such as the normal distribution?</p> <p>Int: De Moivre's derivation of the normal distribution and Quetelet's use of it to describe <i>l'homme moyen</i>.</p>

Topic 6—Core: Calculus

48 hours

The aim of this topic is to introduce students to the basic concepts and techniques of differential and integral calculus and their application.

Content	Further guidance	Links
<p>6.1 Informal ideas of limit, continuity and convergence.</p> <p>Definition of derivative from first principles</p> $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ <p>The derivative interpreted as a gradient function and as a rate of change.</p> <p>Finding equations of tangents and normals.</p> <p>Identifying increasing and decreasing functions.</p> <p>The second derivative.</p> <p>Higher derivatives.</p>	<p>Include result $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$.</p> <p>Link to 1.1.</p> <p>Use of this definition for polynomials only.</p> <p>Link to binomial theorem in 1.3.</p> <p>Both forms of notation, $\frac{dy}{dx}$ and $f'(x)$, for the first derivative.</p> <p>Use of both algebra and technology.</p> <p>Both forms of notation, $\frac{d^2y}{dx^2}$ and $f''(x)$, for the second derivative.</p> <p>Familiarity with the notation $\frac{d^n y}{dx^n}$ and $f^{(n)}(x)$. Link with induction in 1.4.</p>	<p>TOK: The nature of mathematics. Does the fact that Leibniz and Newton came across the calculus at similar times support the argument that mathematics exists prior to its discovery?</p> <p>Int: How the Greeks' distrust of zero meant that Archimedes' work did not lead to calculus.</p> <p>Int: Investigate attempts by Indian mathematicians (500–1000 CE) to explain division by zero.</p> <p>TOK: Mathematics and the knower. What does the dispute between Newton and Leibniz tell us about human emotion and mathematical discovery?</p> <p>Appl: Economics HL 1.5 (theory of the firm); Chemistry SL/HL 11.3.4 (graphical techniques); Physics SL/HL 2.1 (kinematics).</p>

	Content	Further guidance	Links
6.2	<p>Derivatives of x^n, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$.</p> <p>Differentiation of sums and multiples of functions.</p> <p>The product and quotient rules.</p> <p>The chain rule for composite functions.</p> <p>Related rates of change.</p> <p>Implicit differentiation.</p> <p>Derivatives of $\sec x$, $\csc x$, $\cot x$, a^x, $\log_a x$, $\arcsin x$, $\arccos x$ and $\arctan x$.</p>		<p>Appl: Physics HL 2.4 (uniform circular motion); Physics 12.1 (induced electromotive force (emf)).</p> <p>TOK: Mathematics and knowledge claims. Euler was able to make important advances in mathematical analysis before calculus had been put on a solid theoretical foundation by Cauchy and others. However, some work was not possible until after Cauchy's work. What does this tell us about the importance of proof and the nature of mathematics?</p> <p>TOK: Mathematics and the real world. The seemingly abstract concept of calculus allows us to create mathematical models that permit human feats, such as getting a man on the Moon. What does this tell us about the links between mathematical models and physical reality?</p>
6.3	<p>Local maximum and minimum values.</p> <p>Optimization problems.</p> <p>Points of inflexion with zero and non-zero gradients.</p> <p>Graphical behaviour of functions, including the relationship between the graphs of f, f' and f''.</p> <p>Not required: Points of inflexion, where $f''(x)$ is not defined, for example, $y = x^{1/3}$ at $(0,0)$.</p>	<p>Testing for the maximum or minimum using the change of sign of the first derivative and using the sign of the second derivative.</p> <p>Use of the terms “concave up” for $f''(x) > 0$, “concave down” for $f''(x) < 0$.</p> <p>At a point of inflexion, $f''(x) = 0$ and changes sign (concavity change).</p>	

	Content	Further guidance	Links
6.4	<p>Indefinite integration as anti-differentiation.</p> <p>Indefinite integral of x^n, $\sin x$, $\cos x$ and e^x.</p> <p>Other indefinite integrals using the results from 6.2.</p> <p>The composites of any of these with a linear function.</p>	<p>Indefinite integral interpreted as a family of curves.</p> $\int \frac{1}{x} dx = \ln x + c.$ <p>Examples include $\int (2x-1)^5 dx$, $\int \frac{1}{3x+4} dx$ and $\int \frac{1}{x^2+2x+5} dx$.</p>	
6.5	<p>Anti-differentiation with a boundary condition to determine the constant of integration.</p> <p>Definite integrals.</p> <p>Area of the region enclosed by a curve and the x-axis or y-axis in a given interval; areas of regions enclosed by curves.</p> <p>Volumes of revolution about the x-axis or y-axis.</p>	<p>The value of some definite integrals can only be found using technology.</p>	<p>Appl: Industrial design.</p>

	Content	Further guidance	Links
6.6	Kinematic problems involving displacement s , velocity v and acceleration a . Total distance travelled.	$v = \frac{ds}{dt}$, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v \frac{dv}{ds}$. Total distance travelled $= \int_{t_1}^{t_2} v dt$.	Appl: Physics HL 2.1 (kinematics). Int: Does the inclusion of kinematics as core mathematics reflect a particular cultural heritage? Who decides what is mathematics?
6.7	Integration by substitution Integration by parts.	On examination papers, non-standard substitutions will be provided. Link to 6.2. Examples: $\int x \sin x dx$ and $\int \ln x dx$. Repeated integration by parts. Examples: $\int x^2 e^x dx$ and $\int e^x \sin x dx$.	