2. Get to know your exam paper

This chapter will help you to understand the main features of the exam papers, along with some advice on approaching your exams.

General

- Your IB exam consists of two written papers, each worth 40%. The final 20% of your IB mark is from your portfolio.
- Each paper is worth 90 marks and each is 90 minutes long. Both papers are divided into two sections, A and B. Each section is worth 20% of your final mark.

Section A

- Section A consists of short response questions and is intended to assess knowledge and understanding over a broad number of topics on the syllabus.
- A small number of steps will be required to answer each question.
- Questions will not necessarily carry the same mark and you should not assume that equal emphasis will be placed on each syllabus topic.
- Questions will be ordered according to difficulty. That is, questions at the beginning of this section are intended to be easier than those near the end of the section.

Section B

- Section B consists of extended response questions and is intended to assess knowledge and understanding of topics in more depth.
- Each question may require knowledge of more than one topic and may be presented in a variety of forms.
- Questions in this section will also be set in order of increasing difficulty.

Paper 1

- You may hear paper 1 referred to as the "non-GDC paper"; however, throughout this book we will call it "paper 1".
- You are not allowed any type of calculator for this paper. Therefore, the emphasis will be on analytical approaches. For example, are you able to find the equation of a tangent to a curve, or can you multiply matrices without using a GDC? (We highlight some analytical approaches throughout the book.)
- Questions requiring a significant amount of manual calculation will not be set. Marks are awarded for knowledge of the syllabus, not for whether you can multiply three-digit numbers without a calculator.
- Knowledge of all syllabus topics is required for this paper.

Paper 2

- You may hear paper 2 referred to as the "GDC paper"; however, throughout this book we will call it "paper 2".
- For this paper, you must have access to a GDC. Questions will be set that expect you to use a GDC as the primary approach, but not every question will necessarily require its use.
- When approaching questions on this paper, select and **provide evidence** of an appropriate mathematical set-up, then go to your calculator.

- You will normally be rewarded for efficient and effective use of a GDC and should be aware that analytical approaches can often be more tedious and lengthy on this paper. It is up to you to decide which approach is best. The number of marks available for a question can be a very good indicator of how you should proceed.
- There are areas in the syllabus that expect the use of a GDC. For example, if you are asked to find a standard deviation, use your GDC! Valuable time is lost if you do not know when to use your calculator.
- Knowledge of all syllabus topics is required for this paper.

Approaching the exam papers

Study all syllabus topics

- When papers are set, every effort is made to write questions that assess your knowledge of the entire syllabus.
- Re-familiarize yourself with the presumed knowledge. Although questions will not explicitly assess your understanding of this material, you may need to draw upon it to answer questions. For example, you may be expected to apply your knowledge of the trigonometry of right-angled triangles to answer part of a question.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Mathematics is a "pencil sport", so prepare by doing! Although it is important to prepare from past IB exam questions, never assume that your exam will be easier or more difficult than previous exams.

Read the instructions

- On both papers, questions in section A can be answered directly in the spaces provided on the exam paper, while questions in section B must be answered on a separate sheet of paper using a new page for each question.
- Answers on both papers must be given exactly or rounded correctly to three significant figures unless otherwise specified in the question. Failure to do so results in an accuracy penalty. You may lose up to a maximum of one mark on **each** paper for incorrect rounding or accuracy.
- Show your work! You are expected to show your working on **both** papers. Answers given without working may not receive full marks. Incorrect answers with no working are not awarded any marks. Attempts to use a correct method are often awarded method marks, even if you have not carried out the approach correctly.
- Finish your answers! Unfinished answers such as 3/0.1 or $\binom{5}{2}$ will not receive full marks.

How marks are awarded

Later on in the book you will come across marking abbreviations that you may not be familiar with (if you are not familiar with these, check with your teacher). The following table can be used as a guide to help you understand how marks are awarded to students.

M	Marks awarded for attempting to use a correct **method**. Work must be shown to achieve this mark.
A	These are marks awarded for an intermediate and/or final **answer**. They are normally dependent on a preceding M mark. For example, correct substitution into a formula or correctly integrating a function would be awarded A marks.
N	Marks that are awarded for correct answers with **no working**. As we normally expect students to show working, the number of N marks available for a question may be less than the total number of marks available for that question.
R	Marks awarded for clear mathematical **reasoning**.
ft	These are **follow-through** marks awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). To be awarded ft marks, **you must show your work** and not just a final answer based on an incorrect answer to a previous part.
AG	This stands for **answer given** in the question and no marks are awarded. These marks appear mostly in "show that" questions.
AP	An **accuracy penalty** is applied for incorrect rounding, or for answers not given exactly or to three significant figures. This mark can be deducted once per paper.

3. Command the command terms

In this chapter you will gain an understanding of command terms and how they relate to exam questions; these will help you to decide how much work to show when responding to a question.

Command terms are usually action verbs that tell you what to do in a question. They also give you a clue as to how to approach the question and how much working may be expected.

Below is a list of the official IB mathematics SL command terms and their meanings. The list is not exhaustive as some terms may be used that have their usual meaning (for example, "explain", "estimate"), but most questions in the exams use these terms to direct the student in their approach. It is also smart to note mentally the number of marks any question is worth. A "find" question worth 2 marks will require much less working than a "show that" question worth 6 marks.

Take your time to familiarize yourself with these terms—they will be used on exam papers without any explanation of their meaning.

Write down

This means that full marks are awarded for a correct answer to a question and that working does not need to be shown. Little or no calculation will be necessary as often an answer can be known from the information provided. For example:

- The fifth term in the expansion of the binomial $(a + b)^n$ is given by $\binom{10}{4}p^6(2q)^4$.

  Write down the value of n. *[1 mark]*

If you know the pattern of terms in a binomial expansion, simply writing down the answer of "$n = 10$" earns the mark.

Be especially attentive to this command in paper 2 as it is often a clue that you can obtain an answer by using your GDC. Here is an example of a "write down" question from a past paper 2:

- Consider the curve $y = \ln(3x - 1)$. Let P be the point on the curve where $x = 2$.

  Write down the gradient of the curve at P. *[2 marks]*

If all you write down on your paper is "gradient is 0.6" you will earn both marks. You can use the numerical derivative feature of the GDC to find an answer because there is no expectation of any working. If you choose to find $\frac{dy}{dx}$ and substitute $x = 2$, you will still earn both marks if the question is answered correctly. However, any error that leads to a wrong answer earns zero marks. The "write down" command is a clue that such an approach is not required.

Calculate
Find
Determine

These terms mean that any answer found should be accompanied by any relevant working. Typically, this means you need to give the mathematical set-up and then any major steps that lead to a solution. Here is an example from a past paper 1.

- The graph of $y = \sqrt{x}$ between $x = 0$ and $x = a$ is rotated 360° about the x-axis.

  The volume of the solid formed is 32π. **Find** the value of a. *[7 marks]*

Calculate
Find
Determine
(continued)

To gain full marks you should write down a correct integral set-up, $\pi\int_0^a x\,dx = 32\pi$, as this is the main mathematical issue in the question. You should then also include the integration and substitution steps, as these are what lead to the answer of "$a = 8$". If you find this answer and show little or no working, significantly fewer marks will be awarded.

In paper 2, where answers can often be found without any algebraic steps, the required "working" is to give the mathematical set-up that is considered when finding the answer. For example:

- In an arithmetic series, the first term is -7 and the sum of the first 20 terms is 620.

  Find the common difference. *[3 marks]*

Using the formula for an arithmetic series found in the information booklet, you can write the equation used to solve for d, the common difference: $620 = \frac{20}{2}(-14 + 19d)$. No additional working is required as you can now use the GDC to solve and "$d = 4$".

Solve

This means to find the solution or solutions to an equation, sometimes called roots. In paper 1, this will typically involve some algebraic manipulation. For example:

- **Solve** the equation $\ln x - 3 = \ln\frac{1}{x}$, $x > 0$. *[4 marks]*

Your working might look like this:

$\ln x - 3 = \ln 1 - \ln x$

$2\ln x = 3$

$\ln x = \frac{3}{2}$

$x = e^{3/2}$

In paper 2, you are expected to show the mathematical set-up before using the GDC to find a solution. For example:

- Let $\boldsymbol{A} = \begin{pmatrix} 5 & 1 \\ 6 & 2 \end{pmatrix}$, $\boldsymbol{X} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$.

  Solve the matrix equation $\boldsymbol{AX} = \boldsymbol{C}$. *[3 marks]*

Your working might look like this:

$\boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{C}$

$\boldsymbol{X} = \begin{pmatrix} 5 \\ -17 \end{pmatrix}$

Draw

More demanding than the "sketch" command, this term means that you should create an accurate diagram with appropriate labels. You should **draw** to scale and use a straight edge for any straight lines, including axes. Any points should be plotted accurately and joined in a straight line or smooth curve, whichever is appropriate to the diagram. Often such a command is used for drawings such as histograms or box-and-whisker plots, but can also be used for function graphing as well. You should use a pencil to draw a graph or diagram, and it should be done on the square-ruled paper provided for your exam.

Sketch

When asked to sketch a graph, you are expected to hand **sketch** a correct shape with major features in approximately correct positions. These include intercepts, maxima, minima, points of inflexion, asymptotes and, if a domain or range is included, end points.

Plot

This means to mark the position of a point on a diagram.

Deduce

This means to **show** a result using some known information. It is like a "show that" question, and so all relevant working should be given in a complete manner.

Justify

Give a valid mathematical reason for your answer or conclusion. You may be directed to use a particular concept in your **justification**, or it could be left open for you to decide. For example:

- The area of a rectangle, A, is given by $A = 18\sin 2\theta$.

  (a) **Find** $\frac{dA}{d\theta}$. *[2 marks]*

  (b) **Hence**, find the exact value of θ that maximizes the area of the rectangle. *[3 marks]*

  (c) Use the second derivative to **justify** that this value of θ does give a maximum. *[3 marks]*

To **justify** the maximum in part (c), you have to **state** and **show that** $\frac{d^2A}{d\theta^2} < 0$, which means you find the second derivative and substitute the value found in part (b) to obtain a negative value.

Show that

This means that you should show **all** relevant working, often in a "step-by-step" approach, when finding an answer that is given to you. You cannot work backwards in a "show that" question. That is, you cannot use the given answer in your work. Rather, you must show the mathematical steps that achieve it. Continue your working until you reach a stage where the given answer follows obviously and immediately from your last line of work. For example:

- Let $f(x) = e^{x+3}$. **Show that** $f^{-1}(x) = \ln x - 3$. *[2 marks]*

Your working might look like this:

$x = e^{y+3}$

$\ln x = \ln e^{y+3}$

$\ln x = y + 3$

$f^{-1}(x) = \ln x - 3$

If you are unable to show the required result, be sure to use the given answer in subsequent parts.

Hence

This means to use your preceding work to find a required result. If you find a correct answer and do not use the preceding work, fewer or possibly no marks will be awarded. For example:

- The area of a rectangle, A, is given by $A = 18\sin 2\theta$.

  (a) **Find** $\frac{dA}{d\theta}$. *[2 marks]*

  (b) **Hence**, find the exact value of θ that maximizes the area of the rectangle. *[3 marks]*

To answer part (b), it is a requirement that you use your answer from part (a). If you use some other method, perhaps using a graph in the GDC, no marks will be awarded even if your answer is correct.

Hence or otherwise

This means that using the preceding work is the suggested approach to answer the question, but other correct methods can earn full marks as well.

Other useful terms

Some command terms will tell you to perform a specific mathematical operation. To **differentiate** is to find a derivative expression, to **integrate** is to find an integral expression, and to **simplify** is to write your answer in a simple form using arithmetical or algebraic properties. Any other such terms will have their usual mathematical meaning.

4. Top tips

Here are some tips that will help you throughout your exams. Some of them may be obvious, but these are sometimes the ones that students forget!

Before the exam

Be aware of what you can bring into the exam room

- Common tools: pencils, pens, eraser, sharpener, ruler.
- Your GDC and a spare set of batteries.
- Translation dictionary for non-native speakers of the exam language (cannot be electronic).
- Water, but no other food or drinks.
- You are NOT allowed any electronic devices that have wireless capability, QWERTY keyboard functions, or watches with calculator capabilities for paper 2.

In an effort to be fair to all, the GDC must have its memory reset

- The RAM memory must be reset or initialized on all calculators.
- The ROM memory must be reset or initialized on all calculators.

Be familiar with the layout of the information booklet

- Each search for a formula takes time, so you want to know exactly where to go in the booklet for a formula.
- Although it is nice that so much information is given in the booklet, use it as a resource when you really need it and not for every little thing.

Be familiar with IB mathematical notation

- Although there may be various forms of acceptable mathematical notation for any particular concept, only official IB notation will be used in the questions on the exam papers.
- Because this is an international exam, examiners will accept any appropriate and recognizable notation in your c working. For example, the seventh binomial coefficient in the expansion of $(a + b)^{10}$ would be written in the exam paper as $\binom{10}{6}$. But in your own working it is completely acceptable for you to write ${}_{10}C_6$ instead.
- Final answers given in calculator notation that is not mathematical notation will not be accepted. For example, if you obtain an answer in your GDC such as 3.20456E-3, you should write either 3.20×10^{-3} or 0.00320 as your answer.

During the exam

For each paper you get 90 minutes to complete 90 marks of work, so you should expect to spend about a minute of time per mark

- With your 5 minutes of reading time before the exam begins, you should glance through the questions and plan your strategy for which you will do first, which you may skip, and which you will save for the end.
- Although you may feel the pressure of time, try not to rush through the exam. Pace yourself, and try not to spend too long on any one question.

When questions have multiple parts, they are often "linked" in that an answer or concept in one part may be helpful in answering another part

- Section B questions are built around a common theme, so that what you do in the earlier parts of a question may be relevant in later parts.
- If you are stuck for an idea in a later part of a question, say part (c), look at what was asked in parts (a) and (b) for a clue to get you thinking.
- This is especially true in "show that" questions. If you do not know how to show the answer given in the question, don't give up! You can still use the given result in the remainder of the question.

When questions include a diagram, the words of the question will describe the information in the diagram

- Make sure that as you read the words of the question you check the diagram for any given information. Be clear about what information you have and about what you are asked.

It is always smart to check that your answers make sense

- If, for example, you work out a probability that is greater than one, you know something is wrong. It may be worth the effort to go back and find the error, as follow-through marks are not awarded for using unreasonable answers in later parts of a question.
- Other unreasonable answers to look out for include: negative probabilities, sine or cosine greater than one or less than negative one, common ratio of an infinite geometric series greater than one or less than negative one, and square roots of negative numbers. Although square roots of negative numbers do exist in mathematics, they are not examined in mathematics SL.

Cross out any working you do not wish the examiner to mark

- Examiners are instructed to ignore crossed-out working, so you should cross out any unwanted working clearly. A large "X" through the working is sufficient.
- Don't cross out working until you have replacement working. Working that may earn some marks is better than a blank that earns none.
- If the examiner is presented with two sets of working to the same problem, they are told to mark the first that is seen, so there is no advantage to leaving both in place in the hope the best one will be marked.

Paper 2

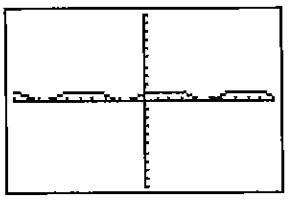
You are expected to be able to use the GDC to perform a variety of special operations, sometimes with functions unfamiliar to you

You should be able to:

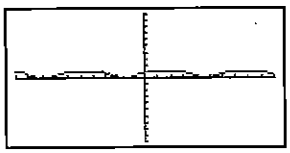
- graph a variety of functions with appropriate windows
- find key features of a graph: y-values, zeros, maxima, minima, gradient, area under a curve, and so on
- solve equations, both graphically and using a solver
- find numerical derivatives and definite integrals
- perform matrix operations (add, subtract, multiply)
- find the determinant and inverse of a 3×3 matrix
- find statistical values for a list of data, with or without frequencies: mean, median, quartiles, standard deviation
- find binomial probabilities, including cumulative probabilities
- find normal probabilities and standardized values (z-scores).

An "appropriate window" is one in which you can see a clear shape of the graph that includes all of its key features. For example, set your GDC to radian mode and consider the graph of $y = \sin(1 + \sin x)$, $0 \le x \le 2\pi$. First see the graph in the standard viewing window, $-10 \le x \le 10$ and $-10 \le y \le 10$.

Texas Instruments



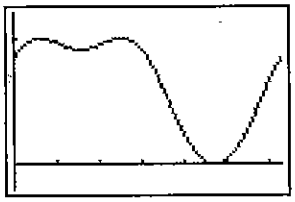
Casio



Between $x = 0$ and about $x = 6$, there appears to be only one minimum and one maximum.

Now look at this same graph using the window $0 \le x \le 2\pi$ and $-0.2 \le y \le 1.2$.

Texas Instruments



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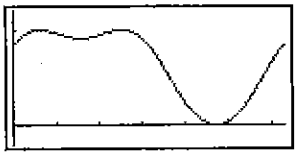

Here you can see how the graph **really** looks in the domain $0 \le x \le 2\pi$. There are clearly **two** minima and **two** maxima.

Remember that the GDC is an effective mathematical tool

- As questions for paper 2 are written with the GDC in mind, then unless it is a "show that" question, consider the GDC in your approach.
- For example, here is a question given on a recent paper 2:

  Solve the equation $e^x = 4\sin x$, for $0 \leq x \leq 2\pi$.

To solve this equation, input $y = e^x$ and $y = 4\sin x$ into the GDC and find the x-coordinates of the intersections on the graphing screen. On your paper give a quick sketch of the two graphs intersecting, and then write your answers. You could also use the solver feature of the GDC, but without a graph you might not see that there are **two** solutions in the given domain. You do **not** want to try to solve this equation with algebraic steps. It can't be done!