INFINITE SUMMATION **SL TYPE I**

Aim: In this task, you will investigate the sum of infinite sequences t_n, where

$$t_0 = 1,\ t_1 = \frac{(x\ln a)}{1},\ t_2 = \frac{(x\ln a)^2}{2\times 1},\ t_3 = \frac{(x\ln a)^3}{3\times 2\times 1} \ldots,\ t_n = \frac{(x\ln a)^n}{n!} \ldots.$$

A notation that you may find helpful in this task is the factorial notation $n!$, defined by

$$n! = n(n-1)(n-2)....3\times 2\times 1 \qquad e.g.\ 5! = 5\times 4\times 3\times 2\times 1\ (=120) \qquad \text{Note that } 0! = 1$$

Consider the following sequence of terms where $x = 1$ and $a = 2$.

$$1, \frac{(\ln 2)}{1}, \frac{(\ln 2)^2}{2\times 1}, \frac{(\ln 2)^3}{3\times 2\times 1} \ldots$$

Calculate the sum S_n of the first n terms of the above sequence for $0 \le n \le 10$. Give your answers correct to six decimal places.

Using technology, plot the relation between S_n and n. Describe what you notice from your plot. What does this suggest about the value of S_n as n approaches ∞?

Consider another sequence of terms where $x = 1$ and $a = 3$.

$$1, \frac{(\ln 3)}{1}, \frac{(\ln 3)^2}{2\times 1}, \frac{(\ln 3)^3}{3\times 2\times 1} \ldots$$

Calculate the sum S_n of the first n terms of this new sequence for $0 \le n \le 10$. Give your answers correct to six decimal places.

Using technology, plot the relation between S_n and n. Describe what you notice from your plot. What does this suggest about the value of S_n as n approaches ∞?

Now consider a general sequence where $x = 1$.

$$1, \frac{(\ln a)}{1}, \frac{(\ln a)^2}{2\times 1}, \frac{(\ln a)^3}{3\times 2\times 1} \ldots$$

Calculate the sum S_n of the first n terms of this general sequence for $0 \le n \le 10$ for different values of a. Give your answers correct to six decimal places.

Using technology, plot the relation between S_n and n. Describe what you notice from your plot. What does this suggest about the value of S_n as n approaches ∞?

Use your observations from these investigations to find a general statement that represents the infinite sum of this general sequence.

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Now we will expand our investigation to determine the sum of the infinite sequence t_n, where

$$t_0 = 1,\ t_1 = \frac{(x \ln a)}{1},\ t_2 = \frac{(x \ln a)^2}{2 \times 1},\ t_3 = \frac{(x \ln a)^3}{3 \times 2 \times 1} \ldots.$$

Define $T_n(a, x)$ as the sum of the first n terms, for various values of a and x,
e.g. $T_9(2, 5)$ is the sum of the first nine terms when $a = 2$ and $x = 5$.

Let $a = 2$. Calculate $T_9(2, x)$ for various positive values of x. Using technology, plot the relation between $T_9(2, x)$ and x. Describe what you notice from your plot.

Let $a = 3$. Calculate $T_9(3, x)$ for various positive values of x. Using technology, plot the relation between $T_9(3, x)$ and x. Describe what you notice from your plot.

Continue with this analysis to find the general statement for $T_n(a, x)$ as n approaches ∞.

Test the validity of the general statement with other values of a and x.

Discuss the scope and/or limitations of the general statement.

Explain how you arrived at the general statement.