



88127302



International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

**MATHEMATICS
STANDARD LEVEL
PAPER 2**



1 hour 30 minutes

Candidate session number

0	0								
---	---	--	--	--	--	--	--	--	--

Examination code

8	8	1	2	-	7	3	0	2
---	---	---	---	---	---	---	---	---

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the *Mathematics SL information booklet* is required for this paper.
- The maximum mark for this examination paper is [90 marks].

AP (3 SF) applied once per paper

UP (failure to write correct unit) applied once per paper



0112

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer all questions in the boxes provided... Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The first three terms of an arithmetic sequence are 5, 6.7, 8.4.

(a) Find the common difference. [2 marks]

(b) Find the 28th term of the sequence. [2 marks]

(c) Find the sum of the first 28 terms. [2 marks]

a) $6.7 - 5 = 8.4 - 6.7 = 1.7$

b) $u_n = u_1 + (n-1)d$
 $= 5 + (n-1)(1.7)$ and
 $u_{28} = 5 + (28-1)(1.7)$
 $= 50.9$

c) $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ or $\frac{n}{2}(u_1 + u_n)$

$S_{28} = \frac{28}{2}(2(5) + 27(1.7))$ or $S_{28} = \frac{28}{2}(5 + (their))$ (Method mark)

$= 14(10 + 45.9)$
 $= 14(55.9)$
 $= 782.6$

$= 14(5 + 50.9)$
 $= 14(55.9)$
 $= 782.6$

2. [Maximum mark: 5]

Let $A = \begin{pmatrix} 2 & 2 & 1 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}$.

- (a) Write down A^{-1} . \Rightarrow GDC as they can not find the inverse of a 3x3 matrix. [2 marks]
- (b) Hence or otherwise, find B , given that $AB = \begin{pmatrix} -1 & 6 & -1 \\ 5 & -1 & 3 \\ 5 & 2 & 7 \end{pmatrix}$. [3 marks]

a) $\begin{pmatrix} -0.5 & 0 & 0.5 \\ 1.5 & 1 & -1.5 \\ -1 & -2 & 2 \end{pmatrix}$

b) $AB = \begin{pmatrix} -1 & 6 & -1 \\ 5 & -1 & 3 \\ 5 & 2 & 7 \end{pmatrix}$ \downarrow left multiply by A^{-1}

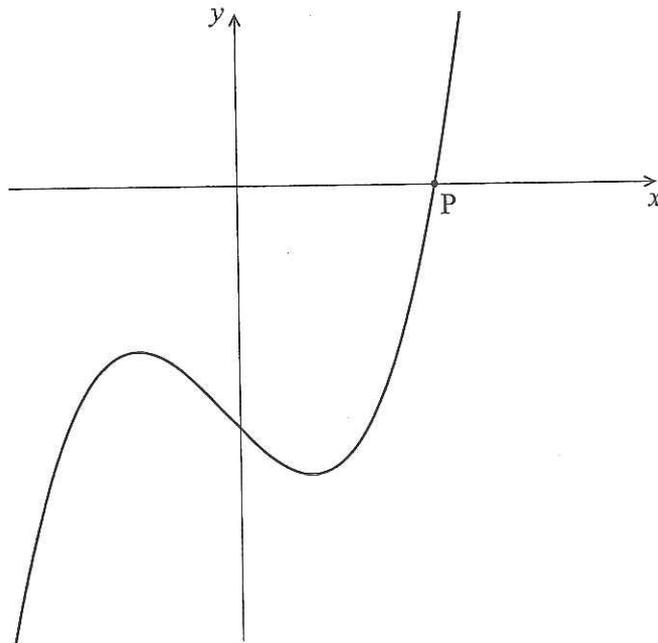
$A^{-1} \cdot A \cdot B = A^{-1} \begin{pmatrix} -1 & 6 & -1 \\ 5 & -1 & 3 \\ 5 & 2 & 7 \end{pmatrix} \Rightarrow$ GDC

$B = \begin{pmatrix} 3 & -2 & 4 \\ -4 & 5 & -9 \\ 1 & 0 & 9 \end{pmatrix}$



3. [Maximum mark: 6]

Let $f(x) = x^3 - 2x - 4$. The following diagram shows part of the curve of f .



The curve crosses the x -axis at the point P.

- (a) Write down the x -coordinate of P. [1 mark]
- (b) Write down the gradient of the curve at P. [2 marks]
- (c) Find the equation of the normal to the curve at P, giving your equation in the form $y = ax + b$. [3 marks]

a) xiat: $y = 0$
 $0 = x^3 - 2x - 4 \Rightarrow$ GDC
 $x = 2$

b) GDC numerical deriv @ $x = 2$ or
 $f'(x) = 3x^2 - 2$
 $f'(2) = 3(2)^2 - 2$
 $= 3 \cdot 4 - 2$
 $= 12 - 2$
 $= 10$

c) \Rightarrow





Candidate session number / Numéro de session du candidat / Número de convocatoria del alumno									
0	0								

Sheet number Feuille n° Hoja núm.		
---	--	--



AB02

ANSWER SHEET
FEUILLE DE RÉPONSES
HOJA DE RESPUESTAS

Please complete the boxes/Veuillez remplir les cases/Llene los recuadros

Question
Question
Pregunta

Examiner
Examinateur
Examinador

c) gradient of tangent @ $\uparrow = 10$
 \therefore gradient of Normal is $-\frac{1}{10}$ (1) and
point of contact is $(2, 0)$.

$\therefore y = ax + b$
 $y = -\frac{1}{10}x + b$
 $0 = -\frac{1}{10}(2) + b$ (1) \leftarrow (2, 0)
 $0 = -\frac{2}{10} + b$
 $\frac{2}{10} = b$
 $b = \frac{1}{5}$

$\therefore y = -\frac{1}{10}x + \frac{1}{5}$ (1)



4. [Maximum mark: 7]

The third term in the expansion of $(2x + p)^6$ is $60x^4$. Find the possible values of p .

Coefficients from pascal or from GOC $\binom{n}{r}$

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \dots$$

$$(2x + p)^6 = (2x)^6 + \binom{6}{1}(2x)^5 \cdot p + \binom{6}{2}(2x)^4 p^2 + \dots$$

$$\binom{6}{2} = 15$$

$$15(2x)^4 p^2 = 60x^4 \quad \text{4 marks for this eqn}$$

$$15(2^4 x^4) p^2 = 60x^4 \quad |$$

$$15 \cdot 16 p^2 = 60$$

$$240 p^2 = 60$$

$$p^2 = \frac{60}{240}$$

$$p^2 = \frac{1}{4} \quad |$$

$$p = \pm \sqrt{\frac{1}{4}}$$

$$= \pm \frac{1}{\sqrt{2}}$$

$$= \pm \frac{2\sqrt{2}}{2}$$

accept any of these (1)

			1			
		1	2	1		
	1	3	3	1		
1	4	6	4	1		
1	6	15	10	5	1	
1	6	15	20	15	6	1

deduct 1 mark if \pm is forgotten.





Candidate session number / Numéro de session du candidat / Número de convocatoria del alumno							
0	0						

Sheet number Feuille n° Hoja núm.		
---	--	--



AB02

ANSWER SHEET
FEUILLE DE RÉPONSES
HOJA DE RESPUESTAS

Please complete the boxes/Veuillez remplir les cases/Llene los recuadros

Question
Question
Pregunta

Examiner
Examinateur
Examinador

d) x-coordinate by hand

$$0 = 5 \cos\left(\frac{\pi}{4}(x-3)\right)$$

$$0 = \cos\left(\frac{\pi}{4}x - \frac{3\pi}{4}\right)$$

$$\therefore \frac{\pi}{4}x - \frac{3\pi}{4} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \text{ etc}$$

$$\frac{4}{\pi} \left(\frac{\pi}{4}x\right) = \left\{ \begin{array}{l} \frac{\pi}{2} + \frac{3\pi}{4} \\ \frac{3\pi}{2} + \frac{3\pi}{4} \\ \frac{5\pi}{2} + \frac{3\pi}{4} \\ \vdots \end{array} \right\} \frac{4}{\pi}$$

$$x = \left\{ \begin{array}{l} \frac{2\pi}{4} + \frac{3\pi}{4} \\ \frac{6\pi}{2} + \frac{3\pi}{4} \\ \frac{10\pi}{2} + \frac{3\pi}{4} \\ \vdots \end{array} \right\} \frac{4}{\pi}$$

$$x = \left\{ \begin{array}{l} 5 \\ 9 \\ 13 \\ \vdots \end{array} \right.$$

(GDC much quicker!)



6. (a) Expand $\sum_{r=4}^7 2^r$ as the sum of four terms.

(1)

(b) (i) Find the value of $\sum_{r=4}^{30} 2^r$.

(ii) Explain why $\sum_{r=4}^{\infty} 2^r$ cannot be evaluated.

(6)

(Total 7 marks)

a) $2^4 + 2^5 + 2^6 + 2^7$ (1)

b) i) $2^4 + 2^5 + 2^6 + \dots + 2^{30}$ $r=2$
 $u_1 = 2^4$
 $u_{27} = 2^{30}$ (1)

$$S_n = \frac{u_1(r^n - 1)}{(r - 1)}$$

$\therefore S_{27} = \frac{2^4(2^{27} - 1)}{(2 - 1)}$ (2) for correct formula & correct substitution

$$= \frac{2^{31} - 2^4}{1}$$

$$= 21,474,832$$
 (1)

ii) The sum is divergent... goes to infinity, only geometric series where $0 < r < 1$ (2)

and S_{∞} be evaluated....

Formula booklet: $S_{\infty} = \frac{u_1}{1-r}$ for $|r| < 1$

but $r = 2$

(anything reasonable)

Radians!

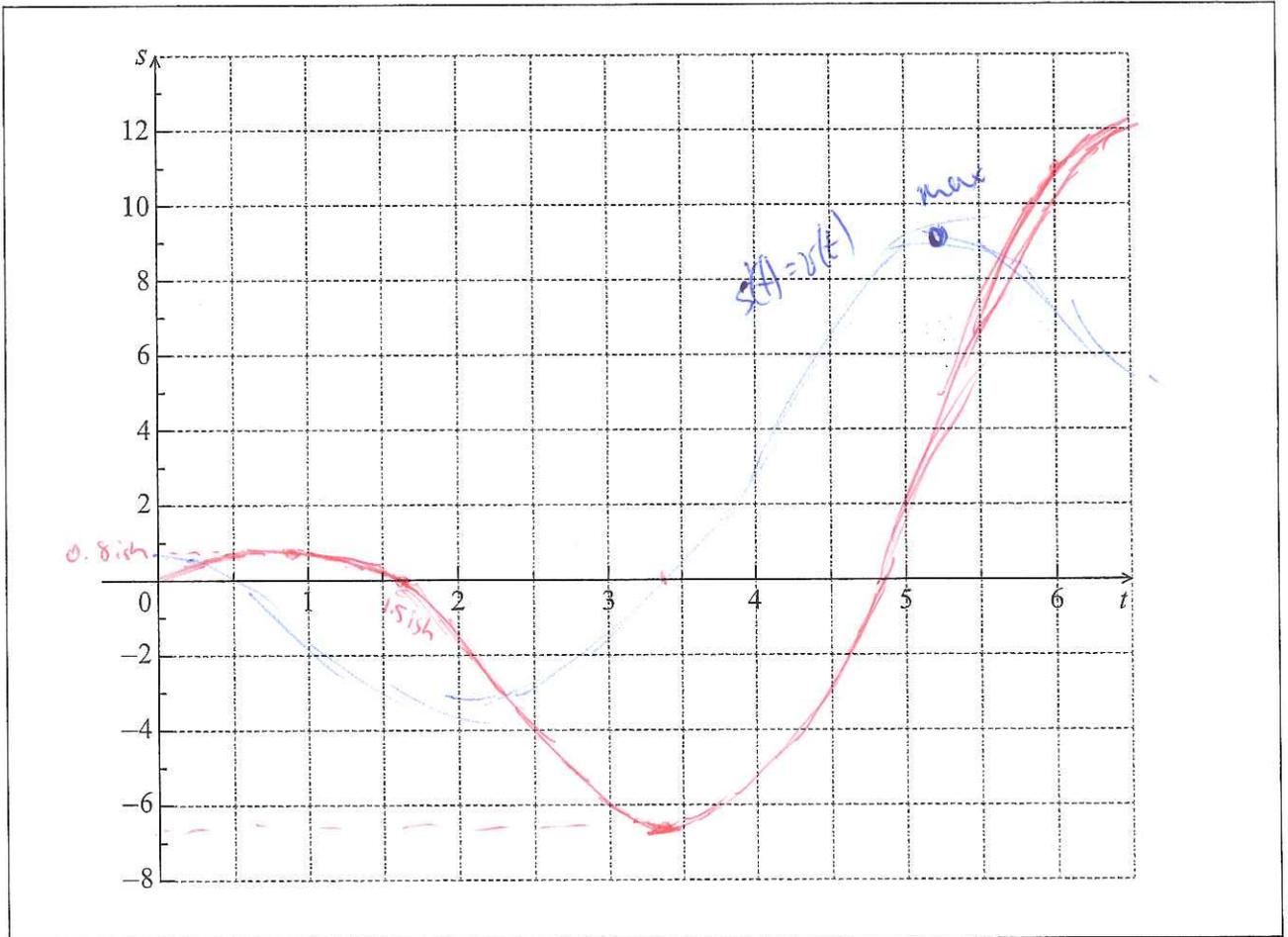
7. [Maximum mark: 7]

A particle's displacement, in metres, is given by $s(t) = 2t \cos t$, for $0 \leq t \leq 6$, where t is the time in seconds.

$\Rightarrow \cos$

(a) On the grid below, sketch the graph of s .

[4 marks]



(intercepts, max + min should be approximately correct. (+0.5))

(This question continues on the following page)



(Question 7 continued)

(b) Find the maximum velocity of the particle.

[3 marks]

$v(t) = s'(t) \rightarrow$ product rule!

$s'(t) = 2t(-\sin t) + (\cos t)(2)$
 $= -2t\sin t + 2\cos t$
 $= 2\cos t - 2t\sin t$ and max when

$a(t) = s''(t) = 0$
 $a(t) = -2\sin t - (2t\cos t + 2\sin t)$
 $= -2\sin t - 2t\cos t - 2\sin t$
 $0 = -2t\cos t - 4\sin t$

$4\sin t = -2t\cos t$
 $\frac{4}{4} \sin t = \frac{-2t}{4} \cos t \Rightarrow x \approx 5.08$
 $v(5.09) \approx 10.2$

Method marks for attempting this.

→ Find max w/ GNC (see previous sketch)

Max when $x \approx 5.0869 \dots$
 $v \approx 10.2002 \dots$
 $v = 10.2$ (3 SF) $\boxed{(m/s)}$

Unit penalty if not included (1 mark)



Do **NOT** write solutions on this page.

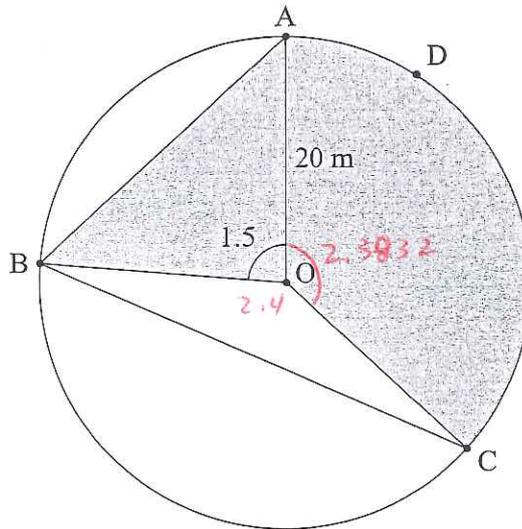
SECTION B

*IMPORTANT to
Read directions!*

Answer **all** questions on the answer sheets provided. Please start each question on a new page.

8. [Maximum mark: 15]

The following diagram shows a circular play area for children.



The circle has centre O and a radius of 20 m, and the points A, B, C and D lie on the circle. Angle AOB is 1.5 radians.

(a) Find the length of the chord [AB]. [3 marks]

(b) Find the area of triangle AOB. [2 marks]

Angle BOC is 2.4 radians.

(c) Find the length of arc ADC. [3 marks]

(d) Find the area of the shaded region. [3 marks]

(e) The shaded region is to be painted red. Red paint is sold in cans which cost \$32 each. One can covers 140 m². How much does it cost to buy the paint? [4 marks]





0

0

Sheet number
Feuille n°
Hoja núm.



AB02

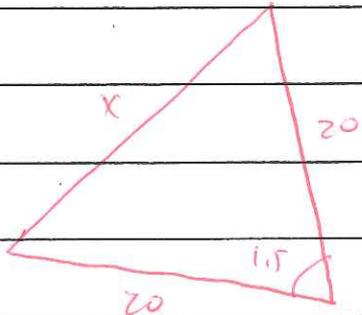
ANSWER SHEET
FEUILLE DE RÉPONSES
HOJA DE RESPUESTAS

Please complete the boxes/Veuillez remplir les cases/Llene los cuadros

Question
Question
Pregunta

Examiner
Examineur
Examinador

a)



$$\therefore x^2 = 20^2 + 20^2 - 2(20)(20)\cos(1.5) \quad (2)$$

$$x^2 \approx 743.41023 \dots$$

$$\therefore x = \pm 27.2655 \dots$$

(1) choose $x \approx 27.3(m)$ unit penalty (3SF)

b) $A = \frac{1}{2} ab \sin \theta$

$$= \frac{1}{2} (20)(20) \sin(1.5)$$

$$\approx 199.49 \dots$$

$$\approx 199(m^2) \quad (3SF) \quad (1)$$

$2\pi - 1.5 - 2.4 \approx 2.3832 \text{ rad}$

Area of sector $A = \frac{1}{2} \theta r^2$

$$= \frac{1}{2} (2.3832)(20)^2 \quad (1)$$

$$\approx 476.64 m^2$$

$\therefore \text{Ashaded} = A_{\Delta} + A_{\Delta}$

depending on their rounding $\approx 199 + 476.64 \quad (M1)$

$$\approx 675.64$$

$$\approx 676 m^2 \quad (3SF) \quad (1)$$




Candidate session number / Numéro de session du candidat / Número de convocatoria del alumno									
0	0								

Sheet number Feuille n° Hoja núm.		
---	--	--



AB02

ANSWER SHEET
FEUILLE DE RÉPONSES
HOJA DE RESPUESTAS

Please complete the boxes/Veuillez remplir les cases/Llene los cuadros

Question
Question
Pregunta

Examiner
Examineur
Examinador

$$d) \quad l = \theta \cdot r \qquad \theta = 2\pi - 1.5 - 2.4$$

$$l = (2.38)(20) \quad (1) \qquad \approx 2.3832 \text{ rad}$$

$$l = 47.6 \text{ (m)} \qquad \approx 2.38 \text{ (3 SF)} \quad (1)$$

e) 1 can = \$32

1 can \Rightarrow 140 m^2 from Area = 676 m^2

$$\text{Cost} = \frac{\$32}{140 \text{ m}^2} \approx \$0.22857 / \text{m}^2$$

$$\therefore \text{for } 676 \text{ m}^2 \Rightarrow 676 \text{ m}^2 \left(\frac{\$32}{140 \text{ m}^2} \right) \approx \$154.5$$

(exact)

OR

$$\frac{676 \text{ m}^2}{140 \text{ m}^2} \approx 4.8$$

$$\frac{154.5}{32} \approx 4.8$$

\therefore 5 cans

\therefore 5 cans minimum

$$= 5(32)$$

$$= \$160$$

Do **NOT** write solutions on this page.

9. [Maximum mark: 15]

Consider the function $f(x) = x^2 - 4x + 1$.

(a) Sketch the graph of f , for $-1 \leq x \leq 5$. [4 marks]

This function can also be written as $f(x) = (x - p)^2 - 3$.

(b) Write down the value of p . [1 mark]

The graph of g is obtained by reflecting the graph of f in the x -axis, followed by a translation of $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$.

(c) Show that $g(x) = -x^2 + 4x + 5$. [4 marks]

The graphs of f and g intersect at two points.

(d) Write down the x -coordinates of these two points. [3 marks]

Let R be the region enclosed by the graphs of f and g .

(e) Find the area of R . [3 marks]



ANSWER SHEET
FEUILLE DE RÉPONSES
HOJA DE RESPUESTAS

Candidate session number / Numéro de session du candidat / Número de convocatoria del alumno									
0	0								
Sheet number Feuille n° Hoja núm.									

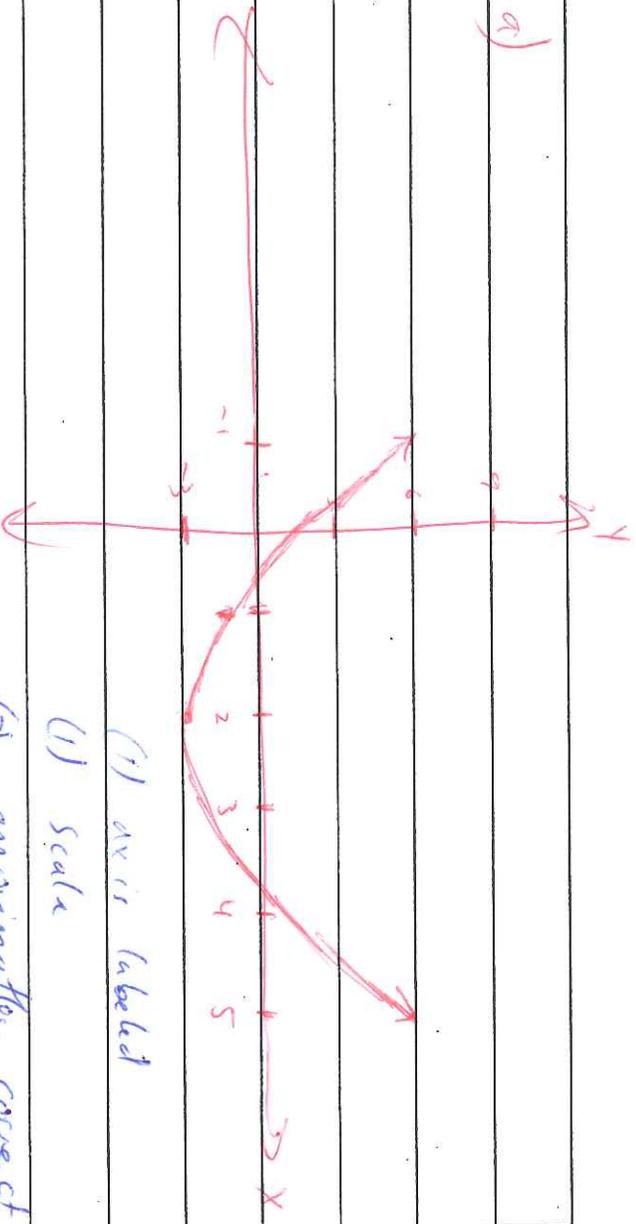


AB02

Please complete the boxes/Veuillez remplir les cases/Llene los recuadros

Question
Question
Pregunta

Examiner
Examinateur
Examinador



(1) axis labeled
(1) scale

(2) approximately correct
min/interrupts

6) $f(x) = (x-2)^2 - 3$

$\therefore p = 2$ (must be +ive)

~~2~~ $-f(x)$ (flips over x-axis)
 $f(x) + 6$ (y shift of 6)

$\therefore g(x) = -f(x) + 6$

~~$= -(x-2)^2 - 3 + 6$~~

~~$= -(x-2)(x-2) + 3$~~

~~$= -(x^2 - 4x + 4) + 3$~~

~~$= -x^2 + 4x - 4 + 3$~~

~~$= -x^2 + 4x - 1$~~



0102

Candidate session number / Numéro de session du candidat /									
Número de convocatoria del alumno									
0	0								

Sheet number Feuille n° Hoja núm.				
---	--	--	--	--



AB02

ANSWER SHEET
FEUILLE DE RÉPONSES
HOJA DE RESPUESTAS

Please complete the boxes/Veuillez remplir les cases/Llene los recuadros

Question
Question
Pregunta

Examiner
Examinateur
Examinador

d) continued

$$g(x) = - (f(x)) + c$$

$$= - ((x-2)^2 - 3) + c \quad M2$$

$$= - (-(x-2)(x-2) + 3) + c \quad M1 \text{ (or there's)}$$

$$= - (x^2 - 4x + 4) + c \quad \text{to simplify}$$

$$= -x^2 + 4x - 4 + c$$

$$= -x^2 + 4x + 5 \quad \leftarrow (1)$$

d) ~~g~~ $-x^2 + 4x + 5 = x^2 - 4x + 1$

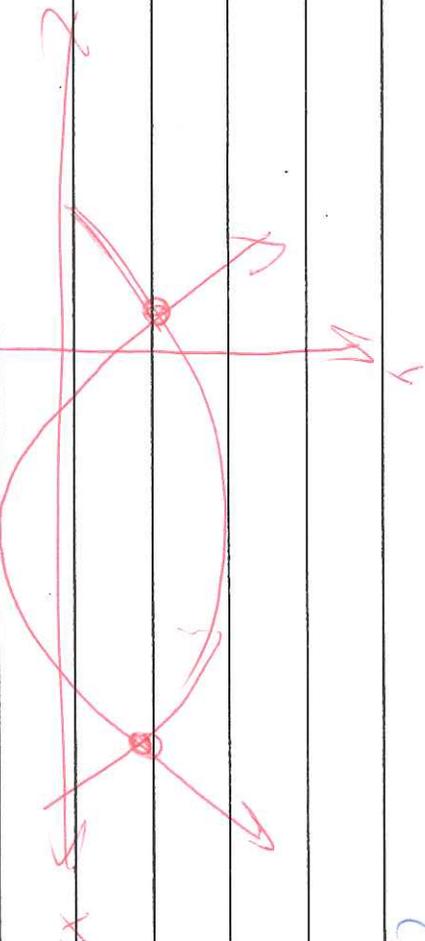
y_1

y_2

\Rightarrow GDC

(should be

supported by a sketch!)



$x \approx -0.44948\dots$ or $x \approx 4.4494\dots$

$x \approx -0.449$ (3SF) $x \approx 4.45$ (3SF)



0102

Candidate session number / Numéro de session du candidat / Número de convocatoria del alumno									
0	0								

Sheet number Feuille n° Hoja núm.				
---	--	--	--	--



AB02

ANSWER SHEET
FEUILLE DE RÉPONSES
HOJA DE RESPUESTAS

Please complete the boxes/Veuillez remplir les cases/Llene los recuadros

Question
Question
Pregunta

Examiner
Examinateur
Examinador

d

$y_u = x^2 + 4x + 1$

$y_l = -x^2 + 4x + 5$

4.445

-0.449

4.445

4.445

$Area \approx 39.1918$

$Area \approx 39.2 \text{ units}^2$

(depending on how they rounding the lower & upper limits)

\Rightarrow GNC

$\int_{-0.449}^{4.445} [(-x^2 + 4x + 5) - (x^2 + 4x + 1)] dx$

(2) for correct statement



0102

10. (*Maximum Mark: 15*)

A curve has equation $y = x(x - 4)^2$.

- (a) For this curve find
- (i) the x -intercepts;
 - (ii) the coordinates of the maximum point;
 - (iii) the coordinates of the point of inflexion.

(9)

- (b) Use your answers to part (a) to sketch a graph of the curve for $0 \leq x \leq 4$, clearly indicating the features you have found in part (a).

(3)

- (c) (i) On your sketch indicate by shading the region whose area is given by the following integral:

$$\int_0^4 x(x - 4)^2 dx.$$

- (ii) Explain, using your answer to part (a), why the value of this integral is greater than 0 but less than 40.

(3)

(Total 15 marks)

Candidate session number / Numéro de session du candidat /									
Número de convocatoria del alumno									
0	0								

Sheet number
Feuille n°
Hoja núm.



AB02

ANSWER SHEET
FEUILLE DE RÉPONSES
HOJA DE RESPUESTAS

Please complete the boxes/Veuillez remplir les cases/Llene los recuadros

Question
Question
Pregunta

Examiner
Examinateur
Examinador

<p>i) xinti: $y = 0$</p> <p>$0 = x(x-4)^2$</p> <p>$\therefore x = 0$ or $x = 4$</p>	(3)
<p>ii) Maximum \Rightarrow GDC or</p> <p>$y = x(x^2 - 8x + 16)$</p> <p>$y = x^3 - 8x^2 + 16x$</p> <p>$\frac{dy}{dx} = 3x^2 - 16x + 16 = 0 \Rightarrow$ GDC</p> <p>$x^2 - 8x + 16 = 0$</p> <p>$(x-4)(x-4) = 0$</p> <p>$x = 4$</p> <p>Maximum @ $(1, 5, 9, 48)$</p>	(3)
<p>iii) Point of inflexion when $\frac{d^2y}{dx^2} = 0$</p> <p>That is, $6x - 16 = 0$</p> <p>$6x = 16$</p> <p>$x = \frac{16}{6}$</p> <p>$x = \frac{8}{3}$</p>	(3)



0102

ANSWER SHEET
FEUILLE DE RÉPONSES
HOJA DE RESPUESTAS

Candidate session number / Numéro de session du candidat /									
Número de convocatoria del alumno									
0	0								
Sheet number Feuille n°									
Hoja núm.									

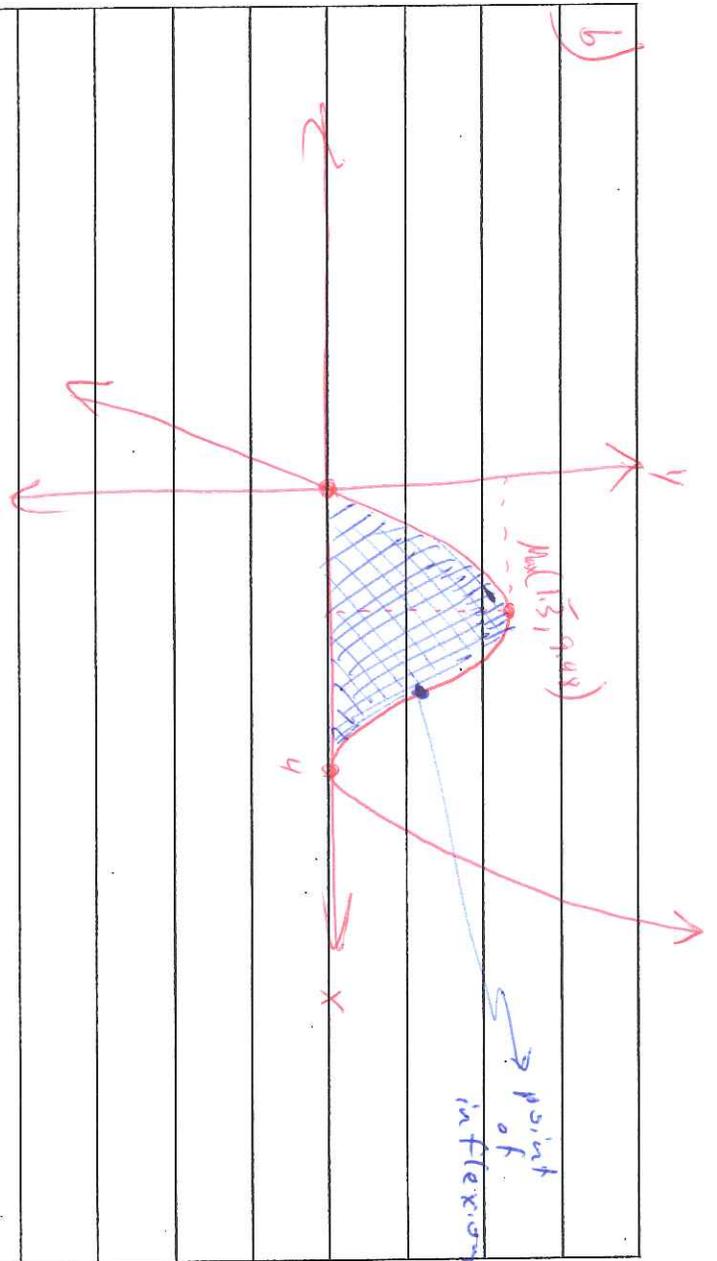


AB02

Please complete the boxes/Veuillez remplir les cases/Llene los recuadros

Question
Question
Pregunta

Examiner
Examinateur
Examinador



c) i) ✓

ii) Some area exist \therefore area > 0 .

A rectangular box 4 units wide + 10 units tall would have an area of 40 units².

Our shape is not even 10 units tall \therefore the area must be less than 40 units².

$0 < y < 10$ and $0 \leq x \leq 4$

So $\int_0^4 y dx < \int_0^4 10 dx \Rightarrow 0 < \int_0^4 y dx < 40$

Words
or
Mathematically



0102