HW

Kinematics:

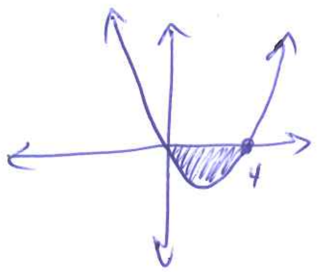
typo ?

1. A Particale starts from rest and moves in a straight line. It velocity at any time t seconds is given by $v(t) = t(t-4)ms^{-1}$. $= t^2-4t$

Find the distance traveled between the two times when the particle is at rest.

The particle is at rest when $V(t) = 0 = t(t-4)$

$\therefore \quad t = 0 \quad$ or $\quad t = 4$



$\therefore$ distance $= \int_0^4 (t^2-4t)\,dt$

$= \left[\frac{t^3}{3} - 2t^2\right]_0^4$

$= \left(\frac{4^3}{3} - 2(4)^2\right) - 0$

$= -\frac{32}{3}$ m $\quad$ but distance is (+) $\therefore \quad \frac{32}{3}$ m,

2. A particle starts from rest and its acceleration, in ms^{-2}, can be modeled by $a(t) = 1-e^{-2t}$, $0 \leq t \leq 3$.

Find the distance traveled in the first 3 seconds.

$a(t) = 1-e^{-2t}$

$\therefore \quad v(t) = \int (1-e^{-2t})\,dt$

$= t - e^{-2t}\left(\frac{1}{-2}\right) + c$

$= t + \frac{1}{2}e^{-2t} + c$ $\qquad$ to get c we know $V(0) = 0$

$0 = 0 + \frac{1}{2}e^0 + c$

$0 = 0 + \frac{1}{2} + c$

$\therefore \quad c = -\frac{1}{2}$

$\therefore \quad v(t) = t + \frac{1}{2}e^{-2t} - \frac{1}{2}$

Now distance $= \int_0^3 \left(t + \frac{1}{2}e^{-2t} - \frac{1}{2}\right)dt$

$= \left[\frac{1}{2}t^2 - \frac{1}{4}e^{-2t} - \frac{1}{2}t\right]_0^3 = 3.25$ m

3. The velocity of a particle moving in a straight line is given by
$v(t) = 10 + 5e^{-0.5t}$ ms^{-1}.

derivative

a.) Show that the acceleration of the particle at any time t is always negative.

$a(t) = 0 + 5e^{-0.5t}(-0.5)$

$= -2.5e^{-0.5t}$

Now $e^{-0.5t} > 0$ (always)

$\therefore$ $-2.5(\text{positive}) = \text{negative}$ (always)

$\therefore$ $a(t) < 0$ for all t.

b.) Find the total distance covered in the first 2 seconds.

$s(t) = \int v(t)$

$\therefore$ distance $= \int_0^2 (10 + 5e^{-0.5t})\,dt$

$= \left[10t - 10e^{-0.5t}\right]_0^2$

$= 26.3m$