MATHEMATICS
STANDARD LEVEL
PAPER 1

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the *Mathematics SL information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

Let $A = \begin{pmatrix} 0 & 3 \\ -2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} -4 & 0 \\ 5 & 1 \end{pmatrix}$.

(a) Find AB. *[3 marks]*

(b) Given that $X - 2A = B$, find X. *[3 marks]*

2.   Let $f(x) = 7 - 2x$ and $g(x) = x + 3$.

(a)   Find $(g \circ f)(x)$.

(2)

(b)   Write down $g^{-1}(x)$.

(1)

(c)   Find $(f \circ g^{-1})(5)$.
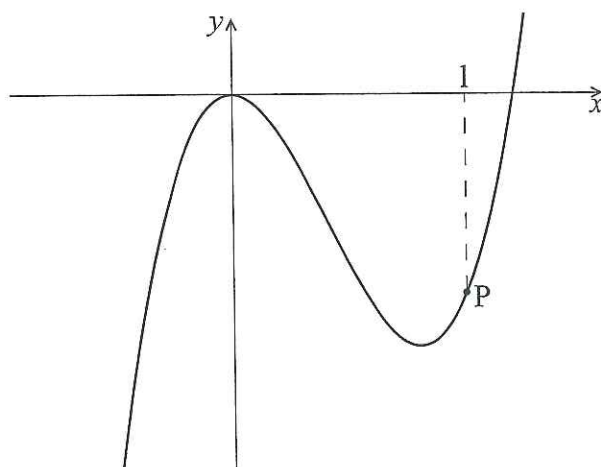
(2)
(Total 5 marks)

3. Solve the equation $2\cos x = \sin 2x$, for $0 \le x \le 3\pi$.

4.   *[Maximum mark: 6]*

Part of the graph of $f(x) = ax^3 - 6x^2$ is shown below.

The point P lies on the graph of f. At P, $x = 1$.

(a)   Find $f'(x)$.   *[2 marks]*

(b)   The graph of f has a gradient of 3 at the point P. Find the value of a.   *[4 marks]*

Turn over

5. *[Maximum mark: 6]*

Let $\sin 100° = m$. Find expressions for each of the following in terms of m

(a) $\cos 100°$; *[3 marks]*

(b) $\tan 100°$; *[1 mark]*

(c) $\sin 200°$. *[2 marks]*

6. *[Maximum mark: 8]*

The line L passes through the point $(5, -4, 10)$ and is parallel to the vector $\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$.

(a) Write down a vector equation for line L.   *[2 marks]*

(b) The line L intersects the x-axis at the point P. Find the x-coordinate of P.   *[6 marks]*

7.   *[Maximum mark: 6]*

The equation $x^2 - 3x + k^2 = 4$ has two distinct real roots. Find the possible values of k.

Please **do not** write on this page.

Answers written on this page
will not be marked.

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Do *NOT* write solutions on this page.

SECTION B

*Answer **all** questions on the answer whets provided.   Please start each question on a new page.*

8.   (*Maximum mark: 16*)

The velocity v m s^{-1} of a particle at time t seconds, is given by $v = 2t + \cos 2t$, for $0 \leq t \leq 2$.

(a)   Write down the velocity of the particle when $t = 0$.

(1)

When $t = k$, the acceleration is zero.

(b)   (i)   Show that $k = \dfrac{\pi}{4}$.

(ii)   Find the exact velocity when $t = \dfrac{\pi}{4}$.

(8)

(c)   When $t < \dfrac{\pi}{4}$, $\dfrac{dv}{dt} > 0$ and when $t > \dfrac{\pi}{4}$, $\dfrac{dv}{dt} > 0$.

Sketch a graph of v against t.

(4)

(d)   Let d be the distance travelled by the particle for $0 \leq t \leq 1$.

(i)   Write down an expression for d.

(ii)   Represent d on your sketch.

(3)
(**Total 16 marks**)

*Do **NOT** write solutions on this page.*

9.   *[Maximum mark: 14]*

Let A and B be points such that $\overrightarrow{OA} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}$.

(a)   Show that $\overrightarrow{AB} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$.   *[1 mark]*

Let C and D be points such that ABCD is a **rectangle**.

(b)   Given that $\overrightarrow{AD} = \begin{pmatrix} 4 \\ p \\ 1 \end{pmatrix}$, show that $p = 3$.   *[4 marks]*

(c)   Find the coordinates of point C.   *[4 marks]*

(d)   Find the area of rectangle ABCD.   *[5 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 16]*

Let $f(x) = \dfrac{6x}{x+1}$, for $x > 0$.

(a) Find $f'(x)$. *[5 marks]*

Let $g(x) = \ln\left(\dfrac{6x}{x+1}\right)$, for $x > 0$.

(b) Show that $g'(x) = \dfrac{1}{x(x+1)}$. *[4 marks]*

(c) Let $h(x) = \dfrac{1}{x(x+1)}$. The area enclosed by the graph of h, the x-axis and the lines $x = \dfrac{1}{5}$ and $x = k$ is $\ln 4$. Given that $k > \dfrac{1}{5}$, find the value of k. *[7 marks]*