

S2

VOLUME OF REVOLUTION INVESTIGATION

STUDENT WORKSHEET

NAME:.....

Setting up your Page

In order to take full advantage of Autograph's unique 3D world, we first need to set up our page correctly. We will do this at the start, and then use the same page for the whole of the investigation.



Open up Autograph in **Advanced Mode**.



Open up a **New 3D Graph Page**.



Make sure you are not in **Whiteboard Mode** (there should not be a blue square around the button).



Change the colour of the background to **Light**.



Change the position of the key to the right-hand side of the screen.



Click on the little arrow next to **x-y-z Orientation**.



Select **x-y Orientation**.



Edit the axes as follows:

x:	Minimum: 0	Maximum: 4	Numbers: 1	Pips: 0.2
y:	Minimum: -12	Maximum: 12	Numbers: 2	Pips: 1
z:	Minimum: -12	Maximum: 12	Numbers: 2	Pips: 1

Remove all of the green ticks underneath **Auto**.

Note: You must ensure all the ticks under **Auto** are removed or Autograph will attempt to re-scale your axes for you.

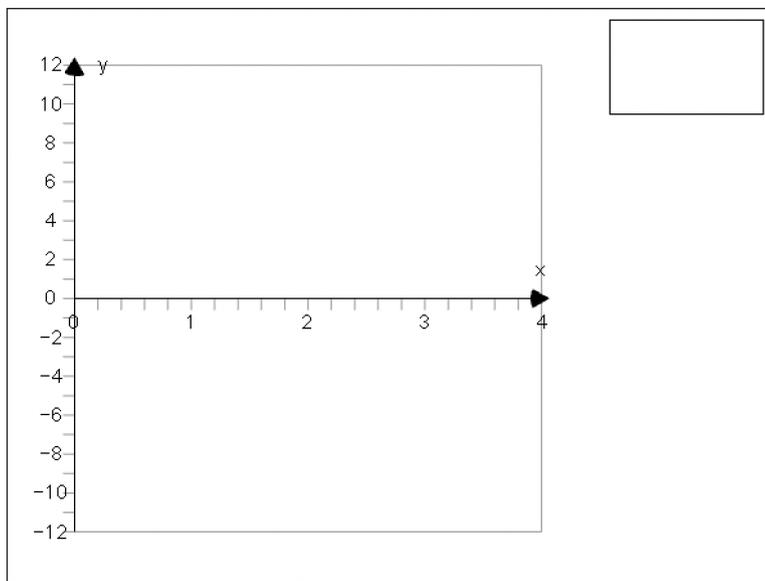


Still on the **Edit Axes** screen, click on the **Options** tab.

Under **Axes**, remove the tick next to **Always Outside**.

Click **OK**.

Your screen should hopefully look something like this:



ACTIVITY 1



Enter the equation: $y = ax + b$

Still on the **Add Equation** screen, place a tick in the space next to **Plot as 2D equation**.

Click on **Edit Constants** and change the value of a to 3 and the value of b to 0.

Predict

What will this line look like? Where will it cross the axes?

Click **OK** twice.



TASK 1: Work out the area of the triangle which would be formed by the x -axis, the line $y = ax + b$, and the line $x = 3$.

Hint: $A = \frac{bh}{2}$



TASK 2: Use integration to work out the same area.

Hint: $A = \int_0^3 3x \, dx$

Checking your answer:



Left-click on the curve (it should turn black).



Enter a co-ordinate with an x value of 0 (leave z unchanged).



Enter another co-ordinate, this time with an x value of 3 (leave z unchanged).

Two points should now be marked on the curve.



Left-click on an unoccupied part of the graph area to *deselect everything*.



Hold down the **Shift** button to select more than one object, then **left-click** on both points (they should both have little squares around them).



Right-click and select **Find Area** from the menu.

Select **Simpson's Rule**, leave the number of divisions at 5, and click **OK**.

In the **View** menu, select **Status Box**, and move it to a convenient position on the page.

The area underneath the line should be displayed here.

Check this matches your answers to Task 1 and Task 2 before moving on.

Note: The **Status Box** only ever displays information about selected objects. If the **Status Box** is not displaying the information you require, simply **left-click** to select the objects again.



TASK 3: The area enclosed by the line $y = ax + b$, the x-axis, and the line $x = 3$ is to be rotated through 360° about the x-axis. Describe the shape of the solid that will be formed.



TASK 4: Find the volume of that solid, leaving your answer in terms of pi (π).

Hint: $V = \pi \int_0^3 (3x)^2 dx$



Checking your answer:



Click on **Slow Plot** (there should be a blue square around the button).



Left-click on the area underneath the curve (it should turn grey).



Right-click and select **Find Volume** from the menu, and click **OK**.

The three-dimensional solid should now begin to form.



Use the **Drag** tool to have a good look around the solid.

Note: You can also use the **Drag** tool as the solid is being rotated.



Press **Undo** a couple of times, and repeat the steps above to view the solid being formed from a different angle.

Note: The following functions may also prove useful when working with Autograph in 3D:



+ **Ctrl** Zooms in and out



+ **Shift** Shifts the camera position



Restores the original view of the graph (**x-y Orientation**).

The volume of the solid should be displayed in the **Status Box**.



If not, simply **left-click** to select the volume (it should turn grey).

Check this matches your answer to Task 4 before moving on.



Select **x-y Orientation** to return to the original view of the graph.



Make sure you are in **Select Mode**.



TASK 5: What other values of a and b would produce a cone with the same volume as this one? Write the values, and the equation of the line these values give us.

Checking your answer:



Click on the **Constant Controller** on the top toolbar.
The **drop-down** menu allows you to select different constants.
The **up-down** arrows alter the value of the selected constant.
The **left-right** arrows adjust the size of the step.



Use the **Constant Controller** to adjust the values of a and b to check your answer to Task 5.

The volume should be displayed in the **Status Box**.



Use the **Drag** tool to have a good look around the solid



TASK 6: Imagine the value of a was 0, and the value of b was 4. Describe the solid that would be formed.



TASK 7: Work out the volume of this solid using integration, and leaving your answer in terms of pi (π).

Hint: What is the equation of the line?

Checking your answer:



Use the **Constant Controller** to adjust the values of a and b to check your answer to Task 7.

The volume should be displayed in the **Status Box**.



Use the **Drag** tool to have a good look around the solid.



Select **x-y Orientation** to return to the original view of the graph



Make sure you are in **Select Mode**.

Change the value of a back to 3, and the value of b to 0.



Left-click on the solid (it should turn white).

Right-click and select **Delete Object** from the menu, or simply press **Delete** on the keyboard.

You should now be left once again with just the area under the line marked on the graph.



TASK 8: Look at the triangle on your screen. Imagine instead that the base of the triangle was not 3, but h, and the height of the triangle was not 9, but r. If this was the case, what would be the equation of the line?

Hint: Think about how to work out the gradient of a straight line.



TASK 9: Use integration to work out the volume of the solid formed when the area underneath this line is rotated 360° about the x-axis.

Hint: $V = \pi \int_0^h y^2 dx$



TASK 10: What is your answer to Task 9 the equation for?

ACTIVITY 2



Close both the **Status Box** and the **Constant Controller** by clicking on the red cross in the corner.

In the **Edit** menu, click on **Select All** and press **Delete**.

You should now be left with a clear set of axes.



Edit the axes as follows:

x: Minimum: -4 Maximum: 4 Numbers: 1 Pips: 0.2

y: Minimum: -4 Maximum: 4 Numbers: 1 Pips: 0.2

z: Minimum: -4 Maximum: 4 Numbers: 1 Pips: 0.2

Remove all of the green ticks underneath **Auto**.



Make sure **Slow Plot** is still on.

Enter the equation: $y = \sqrt{3^2 - x^2}$

Note: To enter x^2 , either use the little  button, or type “**xx**”, or press “**Alt 2**” together.

Still on the **Add Equation** screen, place a tick in the space next to **Plot as 2D equation**.

Predict

What will this line look like? Where will it cross the axes? Will it ever be negative?

Click **OK** twice.



TASK 11: The area enclosed by the line $y = \sqrt{3^2 - x^2}$ and the x-axis is to be rotated through 360° about the x-axis. Describe the shape of the solid that will be formed.



TASK 12: Find the volume of that solid, leaving your answer in terms of pi (π).

Hint: $V = \pi \int_{-3}^3 y^2 dx$, and be careful with negatives!



Checking your answer:



Left-click on the curve (it should turn black).



Enter a co-ordinate with an x value of -3 (leave z unchanged).



Enter another co-ordinate, this time with an x value of 3 (leave z unchanged).

Two points should now be marked on the curve.



Left-click on an unoccupied part of the graph area to *deselect everything*.



Hold down the **Shift** button to select more than one object, then **left-click** on both points (they should both have little squares around them).



Right-click and select **Find Area** from the menu.

Select **Simpson's Rule**, leave the number of divisions at 5 , and click **OK**.



Left-click on the area underneath the curve (it should turn grey).



Right-click and select **Find Volume** from the menu, and click **OK**.



Use the **Drag** tool to have a good look around the solid.



Press **Undo** a couple of times, and repeat the steps above to view the solid being formed from a different angle.

In the **View** menu, select **Status Box**, and move it to a convenient position on the page.

The volume of the solid should be displayed here.

Check this matches your answers to Task 12 before moving on.



Select **x-y Orientation** to return to the original view of the graph.

Make sure you are in **Select Mode**.

Left-click on the solid (it should turn grey).

Right-click and select **Delete Object** from the menu, or simply press **Delete** on the keyboard.

You should now be left once again with just the area under the curve marked on the graph.



TASK 13: Look at the graph on your screen. Imagine that the equation of the curve was actually $y = \sqrt{(r^2 - x^2)}$. Use integration to work out the volume of the solid formed when the area enclosed by this curve and the x-axis is rotated 360° about the x-axis.

Hint: $V = \pi \int_{-r}^r y^2 dx$, and be very careful with negatives!



TASK 14: What is your answer to Task 13 the equation for?

ACTIVITY 3

Close the **Status Box** by clicking on the red cross in the corner.

In the **Edit** menu, click on **Select All** and press **Delete**.

You should now be left with a clear set of axes.



Edit the axes as follows:

x:	Minimum: -20	Maximum: 20	Numbers: 4	Pips: 1
y:	Minimum: -5	Maximum: 5	Numbers: 1	Pips: 0.2
z:	Minimum: -5	Maximum: 5	Numbers: 1	Pips: 0.2

Remove all of the green ticks underneath **Auto**.



Make sure **Slow Plot** is still on.



Enter the equation: $y = \pm\sqrt{x - 2}$

Still on the **Add Equation** screen, place a tick in the space next to **Plot as 2D equation**.

Predict

What will this line look like? Where will it cross the axes? What effect does the \pm have?

Click **OK** twice.



TASK 15: Re-arrange $y = \pm\sqrt{x - 2}$ to make x the subject (i.e. $x =$).



Enter your answer to Task 15 as an equation.

Still on the **Add Equation** screen, place a tick in the space next to **Plot as 2D equation**.

Click **OK**.

The new curve should be plotted on top of the old one.



In the **Key** on the right-hand side of the screen, **left-click** to select Equation 1.



Right-click and select **Delete Current** from the menu.

You should now be left with just the new graph.



TASK 16: Use your answer to Task 15 to help find the area bounded by the curve $y = \pm\sqrt{x - 2}$, the y-axis, and the lines $y = 1$ and $y = 3$.



Checking your answer:



Left-click on the curve (it should turn black).



Enter a co-ordinate with an y value of 1 (leave z unchanged).



Enter another co-ordinate, this time with an y value of 3 (leave z unchanged).

Two points should now be marked on the curve.



Left-click on an unoccupied part of the graph area to *deselect everything*.

Hold down the **Shift** button to select more than one object:



Left-click on both points (they should both have little squares around them).



Right-click and select **Find Area** from the menu.

Select **Simpson's Rule**, leave the number of divisions at 5, and click **OK**.

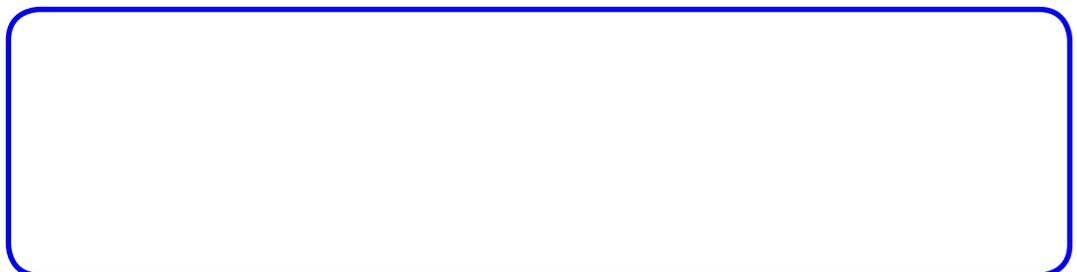
In the **View** menu, select **Status Box**, and move it to a convenient position on the page.

The area to the left of the curve should be displayed here.

Check this matches your answers to Task 16 before moving on.



TASK 17: This area is to be rotated 360° around the y-axis. Briefly describe the shape of the solid that will be formed.





TASK 18: Work out the volume of this solid, leaving your answer in terms of pi (π).

Hint: $V = \pi \int_1^3 x^2 dy$



Checking your answer:



Make sure **Slow Plot** is still on.



Left-click on the area underneath the curve (it should turn grey).



Right-click and select **Find Volume** from the menu, and click **OK**.

The three-dimensional solid should now begin to form.



Use the **Drag** tool to have a good look around the solid.



Press **Undo** a couple of times, and repeat the steps above to view the solid being formed from a different angle.

The volume of the solid should be displayed in the **Status Box**.

Check this matches your answer to Task 18 before moving on.

ACTIVITY 4



Select **x-y Orientation** to return to the original view of the graph.



Make sure you are in **Select Mode**.

Close the **Status Box** by clicking on the red cross in the corner.

In the **Edit** menu, click on **Select All** and press **Delete**.

You should now be left with a clear set of axes.



Edit the axes as follows:

x: Minimum: -2 Maximum: 2 Numbers: 1 Pips: 0.2

y: Minimum: -2 Maximum: 2 Numbers: 1 Pips: 0.2

z: Minimum: -2 Maximum: 2 Numbers: 1 Pips: 0.2

Remove all of the green ticks underneath **Auto**.



TASK 19: Find the point(s) of intersection of $y = x$ and $y = \sqrt{x}$.

Predict

Use your answer to Task 19 to help you predict what these two functions would look like when plotted together.



Make sure **Slow Plot** is still on.



Enter the equation: $y = \sqrt{x}$

Still on the **Add Equation** screen, place a tick in the space next to **Plot as 2D equation**.

Click **OK**.



Enter the equation: $y = x$

Still on the **Add Equation** screen, place a tick in the space next to **Plot as 2D equation**.

Click **OK**.

The graphs of the two functions should now appear on the screen.



TASK 20: Calculate the area that lies between the two functions.

Checking your answer:



Left-click on an unoccupied part of the graph area to *deselect everything*.

Hold down the **Shift** button to select more than one object:



Left-click on the graph of both functions (they should both turn grey).



Right-click and select **Find Area** from the menu.

Select **Simpson's Rule**, change the number of divisions to 40, and click **OK**.

In the **View** menu, select **Status Box**, and move it to a convenient position on the page.

The area enclosed by the two curves should be displayed here.

Check this matches your answer to Task 20 before moving on.



TASK 21: This area is to be rotated 360° about the x-axis. Carefully describe the shape of the solid that will be formed.

Hint: Think about the centre of the object.



TASK 22: Find the volume of that solid, leaving your answer in terms of pi (π).

Checking your answer:



Make sure **Slow Plot** is still on.



Left-click on the area between the curves (it should turn grey).



Right-click and select **Find Volume** from the menu, and click **OK**.

The three-dimensional solid should now begin to form.



Use the **Drag** tool to have a good look around the solid.



Press **Undo** a couple of times, and repeat the steps above to view the solid being formed from a different angle.

The volume of the solid should be displayed in the **Status Box**.

Check this matches your answer to Task 22 before moving on.

ACTIVITY 5 – ONLY FOR THE BRAVE!



Press **Undo** enough times so just the area between the curves is marked on the graph.



Make sure you are in **Select Mode**.

Close the **Status Box** by clicking on the red cross in the corner.



TASK 23: This area is to be rotated 360° about the line $y = -0.5$. Carefully describe the shape of the solid that will be formed.

Hint: Think about the centre of the object.



Make sure **Slow Plot** is still on.



Left-click on the area between the curves (it should turn grey).



Right-click and select **Find Volume** from the menu, change the axis of rotation to $y = -0.5$, and then click **OK**.

The three-dimensional solid should now begin to form.



Use the **Drag** tool to have a good look around the solid.



Press **Undo** a couple of times, and repeat the steps above to view the solid being formed from a different angle.



TASK 24: Show that volume of the solid is exactly equal to $\frac{\pi}{3}$.

Hint: Think about how you could work out the volume of an identical shape, but which is in a different location on the axes.

SUMMARY

Use the space below to summarise what you have learnt during this investigation: