KEY

**Ch. 1/2 Calculator**

**1.** Consider the function $f(x)=\sqrt{2x-1}$

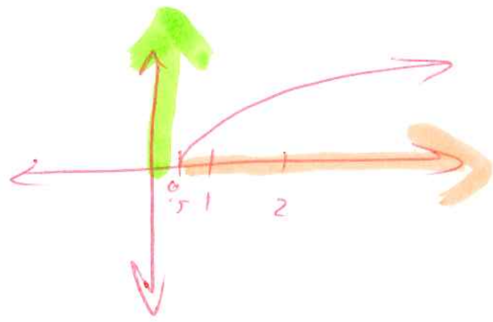
(a) What is the domain of $f(x)$?

$2x-1 \geq 0$

$\therefore\ 2x \geq 1$

$x \geq \frac{1}{2}$

$D: \{x \mid x \in \mathbb{R},\ x \geq \frac{1}{2}\}$




(b) What is the range of *f(x)* ?

$R: \{y / y \in \mathbb{R},\ y \geq 0\}$

**2.** The first three terms of an arithmetic sequence are 8, 11, 14.

$+3\ +3$

$d = 3$

$u_1 = 8$

(a) What is the 41st term of the sequence?

$u_n = u_1 + (n-1)d$

$\therefore\ u_n = 8 + (n-1)(3)$

$= 8 + 3n - 3$

$u_n = 3n + 5$

$\therefore\ u_{41} = 3(41) + 5$

$= 128$

(b) What is the sum of the first 101 terms of the sequence?

$S_n = \frac{n}{2}(2u_1 + (n-1)d)$

$S_{101} = \frac{101}{2}\left(2 \cdot 8 + (101-1)3\right)$

$= 50.5(16 + 100(3))$

$= 50.5(316)$

$= 15{,}958$

**3.** A company offers its employees a choice of two salary schemes A and B over a period of 5 years.

Scheme A offers a starting salary of \$8 000 in the first year and then an annual increase of \$450 per year.

(a) (i) Write down the salary paid in the second year and in the third year.

| Year 1 | Year 2 | Year 3 |
|---|---|---|
| 8,000 | 8450 | 8900 |

(ii) Calculate the **total** (amount of) salary paid over five years.

$S_5 = 8000 + 8450 + 8900 + 9350 + 9800 = 44,500$

or $S_5 = \frac{5}{2}(2 \cdot 8000 + (5-1)450)$

Scheme B offers a starting salary of \$15 000 dollars in the first year and then an annual increase of 5.5 % of the previous year's salary.

(b) (i) Write down the salary paid in the second year and in the third year.

$r = 100\% + 5.5\%$
$= 1 + 0.055$
$= 1.055$

| Year 1 | Year 2 | Year 3 |
|---|---|---|
| 15,000 | $15,000(1.055)^1 = 15825$ | $15,000(1.055)^2 = 16,695.38$ (2 D.P.) |

(ii) Calculate the salary paid in the tenth year.

$U_n = U_1 \cdot r^{n-1}$ $\Longrightarrow$ Note: $U_1 = 15,000(1.055)^{1-1}$
$= 15,000(1.055)^0$
$= 15,000$

$\therefore U_{10} = 15,000 \cdot (1.055)^{10-1}$
$= 24,286.41$ (2 D.P.)

Ch.1/2 Non calc

**1.** Two functions $f$, $g$ are defined as follows:

$$f : x \to 5x - 5$$
$$g : x \to 2(1 - 2x)$$

Find:

(a) $f(2)$ $= 5(2) - 5$
$= 10 - 5$
$= 5$

(b) $(g \circ f)(-4)$

$g(f(x)) = 2(1 - 2(f(x)))$
$g(-25) = 2(1 - 2(-25))$
$= 2(1 + 50)$
$= 2(51)$
$= 102$

$f(-4) = 5(-4) - 5$
$= -20 - 5$
$= -25$

**2.** Consider the functions $f : x \mapsto 2(x - 3)$ and $g : x \mapsto \dfrac{5+x}{2}$.

(a) Find $g^{-1}(x)$

$y = \dfrac{5+x}{2}$

$x = \dfrac{5+y}{2}$

$2x = 5 + y$
$y = 2x - 5 = g^{-1}(x)$

(c) Solve the equation $(f \circ g^{-1})(x) = 4$.

$f(g^{-1}(x)) = 4$
$2((2x-5) - 3) = 4$
$2(2x - 8) = 4$
$4x - 16 = 4$
$4x = 20$ $\therefore x = 5$

**3.** Express in simplest form with a prime number base.

(a) $\dfrac{9^{2x}}{27^{y}}$ $= \dfrac{(3^2)^{2x}}{(3^3)^y} = \dfrac{3^{4x}}{3^{3y}} = 3^{4x-3y}$

(b) $\dfrac{1}{16^{b}}$. $= \dfrac{1}{(2^4)^b} = \dfrac{1}{2^{4b}} = 2^{-4b}$

**4.** Find the sum of the arithmetic series

$27 + 35 + 33 + \ldots + 857.$

$\underbrace{27 + 35}_{8} + \underbrace{35 + 33}_{8}$

$\therefore d = 8$

$u_1 = 27$

$u_n = \boxed{859 = u_1 + (n-1)d}$

$859 = 27 + (n-1)8$

$832 = (n-1)8$

$832 = 8n - 8$

$840 = 8n$

$105 = n$

$\therefore S_n = \frac{n}{2}(u_1 + u_{105})$

$S_{105} = \frac{105}{2}(27 + 859)$

$= 46{,}515$

**5.** Find the sum of the infinite geometric series

$$\frac{2}{3}-\frac{4}{9}+\frac{8}{27}-\frac{16}{81}+\dots$$

$$r=\frac{-\frac{4}{9}}{\frac{2}{3}}=\frac{\frac{8}{27}}{-\frac{4}{9}}$$

$$=\frac{-4}{9}\cdot\frac{3}{2}=-\frac{2}{3}$$

$$S=\frac{u_1}{1-r}$$

$$=\frac{2/3}{1-(-2/3)}$$

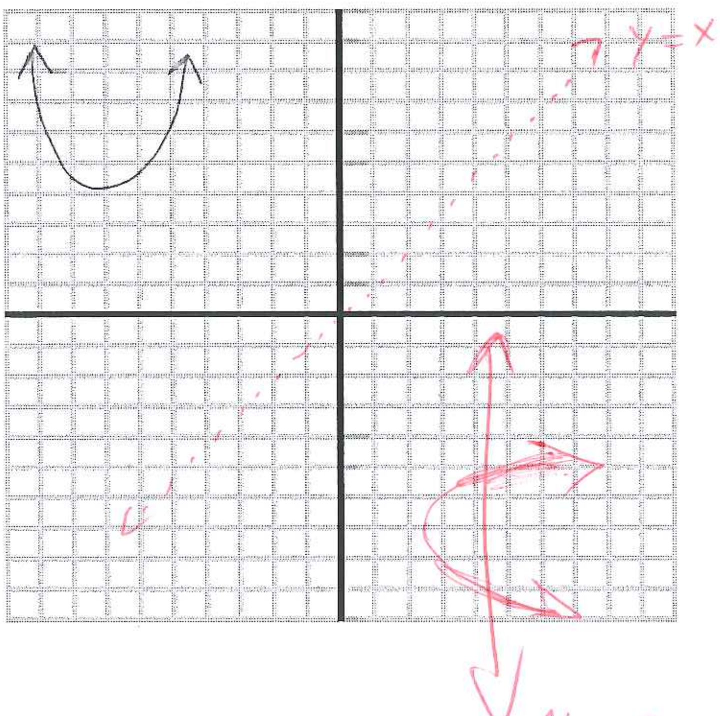
$$=\frac{2/3}{1+2/3}$$

$$=\frac{2/3}{\frac{3}{3}+\frac{2}{3}}$$

$$=\frac{2/3}{5/3}$$

$$=\frac{2}{3}\cdot\frac{3}{5}=\frac{2}{5}\,!$$

**6.** Use the graph of the following function to sketch the graph of the inverse function. Is the inverse a function? (Not a function)




No function!

**ch. ¾ Non calc**

**1.** Solve the equation $27^{x-1} = \left(\frac{1}{3}\right)^{2x}$.

$$(3^3)^{x-1} = (3^{-1})^{2x}$$

$$3^{3(x-1)} = 3^{-1(2x)}$$

$$\therefore 3(x-1) = -1(2x)$$

$$3x-3 = -2x$$

$$-3 = -2x-3x$$

$$\frac{-3}{-5} = \frac{-5x}{-5}$$

$$x = +3/5$$

**2.** Solve $\log_{25}\left(\sqrt{x^2-24}\right) = \frac{1}{2}$

$\Longleftrightarrow$

$$25^{1/2} = \sqrt{x^2-24}$$

$$\sqrt{25} = \sqrt{x^2-24} \quad \longrightarrow \quad 25 = x^2-24$$

$$5 = \sqrt{x^2-24}$$

$$5^2 = \left(\sqrt{x^2-24}\right)^2$$

$$5^2 = x^2-24$$

$$25 = x^2-24$$

$$\sqrt{49} = \sqrt{x^2}$$

$$x = \pm 7$$

**3.** Let $a = \log x$, $b = \log y$, and $c = \log z$.

Write $\log\left(\dfrac{x^2\sqrt{y}}{z^3}\right)$ in terms of $a$, $b$ and $c$. Use the log rules Luke!

$$= \log(x^2\sqrt{y}) - \log(z^3)$$

$$= \log(x^2) + \log(\sqrt{y}) - \log(z^3)$$

$$= 2\log(x) + \log(y^{1/2}) - 3\log(z)$$

$$= 2\log(x) + \frac{1}{2}\log(y) - 3\log(z)$$

$$= 2a + \frac{1}{2}b - 3z$$

**4.** (a) Given that $\log_3 x + \log_3(x - 5) = \log_3 A$, express $A$ in terms of $x$.

$$\log_3(x \cdot (x-5)) = \log_3(A)$$

$$\therefore A = x(x-5)$$

(b) Hence or otherwise, solve the equation $\log_3 x + \log_3(x - 5) = 14$.

$$\log_3(A) = 14 \iff 3^{14} = A$$

$$3^{14} = x^2 - 5x$$

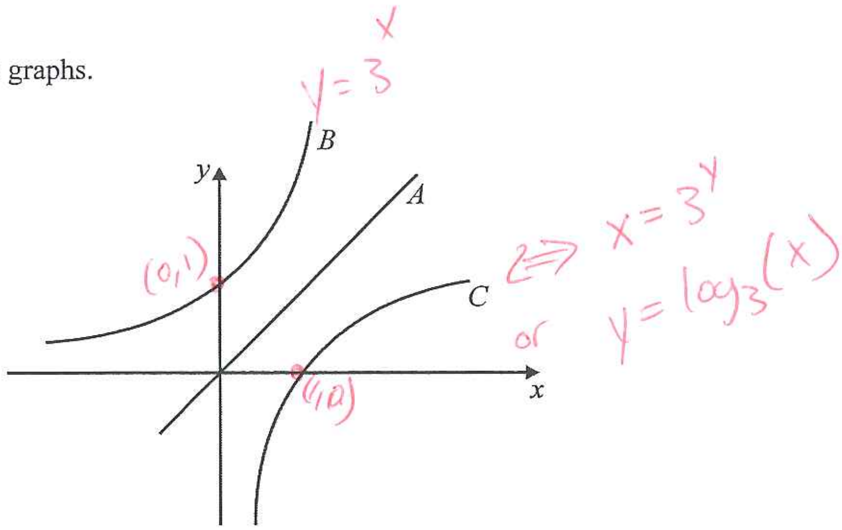
$$0 = x^2 - 5x - 3^{14}$$

$$a = 1$$
$$b = -5$$
$$c = -3^{14}$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{25 - 4(1)(-3^{14})}}{2(1)} \quad \ldots \text{good enough for non-calc!}$$

**5.** The diagram shows three graphs.




$A$ is part of the graph of $y = x$.

$B$ is part of the graph of $y = 3^x$.

$C$ is the reflection of graph $B$ in line $A$.

Write down

(a) the equation of $C$ in the form $y = f(x)$;

$y = \log_3(x)$

(b) the coordinates of the point where $C$ cuts the $x$-axis.

$(1, 0)$

Ch.3/4 Calculator

**1.** The mass $m$ kg of a radio-active substance at time $t$ hours is given by

$$m = 4e^{-0.4t}.$$

(a) Write down the initial mass.

$m = 4 \quad (t=0)$

(b) The mass is reduced to 0.75 kg. How long does this take?

$$\underbrace{0.75}_{Y_1} = \underbrace{4e^{-0.4t}}_{Y_2} \qquad \text{or} \ldots$$

$$\frac{0.75}{4} = e^{-0.4t}$$

$$\ln\left(\frac{0.75}{4}\right) = -0.4t \cdot \cancel{\ln(e)} \; 1$$

$$t = \frac{\ln\left(\frac{0.75}{4}\right)}{-0.4} \simeq 4.18 \text{ hours}$$

**2.** \$1000 is invested at 5% per annum interest, **compounded monthly**. Calculate the minimum number of months required for the value of the investment to exceed \$2000.

$$A = a_1\left(1 + \frac{r}{k}\right)^{kn}$$

$$\underbrace{2000}_{Y_1} = \underbrace{1000\left(1 + \frac{0.05}{12}\right)^{12n}}_{Y_2}$$

Or...

$$\frac{2000}{1000} = (1.0041\bar{6})^{12n}$$

$$\log(2) = \log(1.0041\bar{6})^{12n}$$

$$\frac{\log(2)}{12\log(1.0041\bar{6})} = \frac{12n \cdot \log(1.0041\bar{6})}{12 \log(1.0041\bar{6})}$$

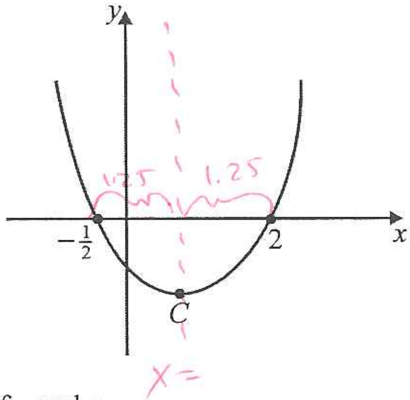
$$n \simeq 1.415 \text{ years} \quad \times 12$$

$$\simeq 16.97 \quad \therefore 17 \text{ months } \underline{\text{minimum}}$$

# ch.5/6 NC

**1.** The diagram represents the graph of the function

$$f : x \mapsto (x-p)(x-q).$$




(a) Write down the values of $p$ and $q$.

$p = -\frac{1}{2}$

$q = 2$

(careful w/ + and −)

$\therefore f(x) = \underbrace{\left(x+\frac{1}{2}\right)}_{\text{0 when } x = -\frac{1}{2}}\underbrace{(x-2)}_{\text{0 when } x = 2}$

(b) The function has a minimum value at the point $C$. Find the equation for the axis of symmetery.


$\therefore \frac{1}{2}$ way $= 1.25$

$x = 0.75$

**2.** The quadratic equation $kx^2 + 6x + 1.5 = 0$, $k > 0$ has no real solutions for $x$.
Find the value of $k$.

$\therefore \Delta = b^2 - 4ac < 0$

$a = k$

$b = 6$

$c = 1.5$

$\therefore \Delta = 6^2 - 4(k)\left(\frac{3}{2}\right) < 0$

$= 36 - 6k < 0$

$36 < 6k$

$6 < k$

$\therefore k > 6$

**1.** (a) Express $f(x) = x^2 - 8x + 24$ in the form $f(x) = (x - h)^2 + k$, where $h$ and $k$ are to be determined.

$x^2 - 8x + 24 = 0$

$x^2 - 8x + 16 = -24 + 16$

$(x-4)^2 = -8$

$(x-4)^2 + 8 = 0$

$\therefore\ y = (x-4)^2 + 8$

$+ \left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

(b) Hence, or otherwise, write down the coordinates of the vertex of the parabola with equation $y = x^2 - 8x + 24$.

GDC or $V(4, 8)$

~~2. Find the equation of the perpendicular bisector for the line segment joining the points (1, 5) to (7, 3).~~

**3.** The quadratic function $f$ is defined by $f(x) = 3x^2 - 18x + 14$.

(a) Write $f$ in the form $f(x) = 3(x - h)^2 - k$.

$$f(x) = 3\left(x^2 - 6x + \frac{14}{3}\right)$$

$$x^2 + -6x + \frac{14}{3} = 0 \qquad \left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$$

$$x^2 - 6x + 9 = -\frac{14}{3} + 9$$

$$(x-3)^2 = \frac{-14}{3} + \frac{27}{3}$$

$$(x-3)^2 = \frac{13}{3}$$

$$(x-3)^2 + \frac{13}{3} = 0$$

$$\therefore f(x) = 3\left((x-3)^2 + \frac{13}{3}\right) = 3(x-3)^2 + 13$$

(b) The graph of $f$ is translated 3 units in the positive $x$-direction and 5 units in the positive $y$-direction. Find the function $g$ for the translated graph, giving your answer in the form $g(x) = 3(x - p)^2 + q$.

$$g(x) = 3((x-3)-3)^2 + 5 + 13$$

$$= 3(x-6)^2 + 18$$

~~**4.** Find $a$ if the distance between $(a, 2)$ and $(8, 10)$ is 10 units.~~