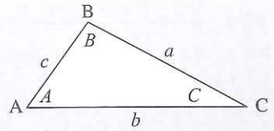
For the triangle alongside:



Area formula Area $= \frac{1}{2}ab\sin C$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

Sine rule $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

If you have the choice of rules to use, use the cosine rule to avoid the **ambiguous case**.

THE GENERAL SINE FUNCTION

If we begin with $y = \sin x$, we can perform transformations to produce the **general sine function** $f(x) = A\sin B(x-C)+D$.

We have a vertical stretch with factor A and a horizontal stretch with factor $\frac{1}{B}$, followed by a translation with vector $\begin{pmatrix} C \\ D \end{pmatrix}$.

The general sine function has the following properties:

- the **amplitude** is $|A|$
- the **principal axis** is $y = D$
- the **period** is $\frac{2\pi}{B}$.

OTHER TRIGONOMETRIC FUNCTIONS

$y = \cos nx$ has period $\frac{2\pi}{n}$.

$y = \tan nx$ has period $\frac{\pi}{n}$.

cosecant x or $\csc x = \dfrac{1}{\sin x}$. secant x or $\sec x = \dfrac{1}{\cos x}$.

cotangent x or $\cot x = \dfrac{1}{\tan x} = \dfrac{\cos x}{\sin x}$.

When graphing $\csc x$, $\sec x$ and $\cot x$, there will be vertical asymptotes corresponding to the zeros of $\sin x$, $\cos x$ and $\tan x$. $\cot x$ will have zeros corresponding to the vertical asymptotes of $\tan x$.

TRIGONOMETRIC IDENTITIES

$\cos^2\theta + \sin^2\theta = 1$

$\cos(\theta + 2k\pi) = \cos\theta$ and $\sin(\theta + 2k\pi) = \sin\theta$ for all $k \in \mathbb{Z}$

$\cos(-\theta) = \cos\theta$, $\sin(-\theta) = -\sin\theta$ and $\tan(-\theta) = -\tan\theta$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ and $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

$\tan^2 x + 1 = \sec^2 x$ and $1 + \cot^2 x = \csc^2 x$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

$\sin 2A = 2\sin A\cos A$

$\cos 2A = \begin{cases} \cos^2 A - \sin^2 A \\ 1 - 2\sin^2 A \\ 2\cos^2 A - 1 \end{cases}$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos(2x)$

$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x)$

TRIGONOMETRIC EQUATIONS

To solve trigonometric equations we can either use graphs from technology, or algebraic methods involving the trigonometric identities. In either case we must make sure to include all solutions on the specified domain.

We need to use the inverse trigonometric functions to invert sin, cos and tan.

Function	*Domain*	*Range*
$x \mapsto \arcsin x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$x \mapsto \arccos x$	$[-1, 1]$	$[0, \pi]$
$x \mapsto \arctan x$	$]-\infty, \infty[$	$\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$

The ranges of these functions are important because our calculator will only give us the one answer in the range. Sometimes other solutions may also be possible. For example, when using arcsin our calculator will always give us an acute angle answer, but the obtuse angle with the same sine may also be valid.

An equation of the form $a\sin x = b\cos x$ can always be solved as $\tan x = \frac{b}{a}$.

SKILL BUILDER QUESTIONS

1 Convert:

a $\frac{2\pi}{9}$ radians to degrees **b** 140° to radians.

2 Find the exact value of:

a $\sin\left(\frac{5\pi}{3}\right)$ **b** $\cos\left(\frac{3\pi}{4}\right)$ **c** $\tan\left(-\frac{\pi}{3}\right)$

3 A sector of a circle of radius 10 cm has a perimeter of 40 cm. Find the area of the sector.

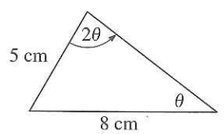
4 A sector of a circle has an arc length of 6 cm and an area of 20 cm^2. Find the angle of the sector.

5 A chord of a circle has length 6 cm. If the radius of the circle is 5 cm, find the area of the minor segment cut off by the chord.

6 Find the largest angle of the triangle with sides 11 cm, 9 cm and 7 cm.

7 In triangle ABC, AB = 15 cm, AC = 12 cm and angle ABC measures 30°. Find the size of the angle ACB.

8



a Find $\cos\theta$. **b** Find the area of the triangle.

9 In triangle PQR, PR = 12 cm, RQ = 11 cm, and $\widehat{RPQ} = 60^o$. Find the length of [PQ], giving your answer in radical form.

10 What consecutive transformations map the graph of $y = \sin x$ onto:

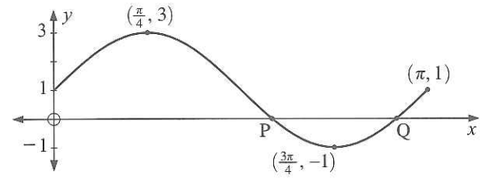
a $y = 2\sin\left(\frac{x}{3}\right)$ **b** $y = \sin\left(x + \frac{\pi}{3}\right) - 4$?

11 Find the period of:

a $y = -\sin(3x)$ **b** $y = 2\sin\left(\frac{x}{2}\right) + 1$

c $y = \sin^2 x + 5$.

12



For the illustrated sine function, find the coordinates of the points P and Q.

13 On the same set of axes, sketch the graphs of $f(x) = \sin x$ and $g(x) = -1 + 2f(2x + \frac{\pi}{2})$ for $-\pi \leqslant x \leqslant \pi$.

14 Find the amplitude, principal axis and period of the following functions:

a $f(x) = \sin 4x$ **b** $f(x) = -2\sin\left(\frac{x}{2}\right) - 1$.

15 Find the period of:

a $y = \cos\left(\frac{x}{3}\right)$ **b** $y = \tan(5x)$

c $y = \sin 3x + \sin x$.

16 Find the exact period of $g(x) = \tan 2x + \tan 3x$.

17 Find the equations of the vertical asymptotes on $[-2\pi, 2\pi]$ for:

a $f(x) = \csc(x)$ **b** $f : x \mapsto \sec(2x)$

c $g : x \mapsto \cot\left(\frac{x}{2}\right)$.

18 Sketch the graph of $y = \csc(x)$ for $x \in [0, 3\pi]$.

19 Sketch the graph of $y = \arccos x$, clearly showing the axes intercepts and endpoints.

20 Find the exact value of $\arcsin(-\frac{1}{2}) + \arctan(1) + \arccos(-\frac{1}{2})$.

21 Find:

a $\sin\left(\arccos(-\frac{\sqrt{3}}{2})\right)$ **b** $\tan\left(\arcsin(\frac{1}{\sqrt{2}})\right)$

22 Simplify $\sin\left(\frac{3\pi}{2} - \phi\right)\tan(\phi + \pi)$.

23 Simplify $1 - \dfrac{\sin^2\theta}{1 + \cos\theta}$.

24 Find the exact value of $\cos 79^o\cos 71^o - \sin 79^o\sin 71^o$.

25 If $\cos(2x) = \frac{5}{8}$, find the exact value of $\sin x$.

26 θ is obtuse and $\sin\theta = \frac{2}{3}$. Find the exact value of $\sin 2\theta$.

27 If $\tan\theta = 2$, find the exact values of $\tan 2\theta$ and $\tan 3\theta$.

28 Given that $\tan 2A = \sin A$ where $\sin A \neq 0$, find $\cos A$ in simplest radical form.

29 Suppose $\sin x - 2\cos x = A\sin(x + \alpha)$ where $A > 0$ and $0 < \alpha < 2\pi$. Find A and α.

30 If $2\theta \in [\pi, \frac{3\pi}{2}]$ and $\tan(2\theta) = 2$, find the exact value of $\tan\theta$.

31 If $\sin x = 2\sin\left(x - \frac{\pi}{6}\right)$, find the exact value of $\tan x$.

32 Show that $\csc(2x) - \cot(2x) = \tan x$ and hence find the exact value of $\tan\left(\frac{5\pi}{12}\right)$.

33 Show that $\dfrac{1}{\tan\theta - \sec\theta} = -(\sec\theta + \tan\theta)$ provided that $\cos\theta \neq 0$.

34 Solve $\sin 2x = \sin x$ for $x \in [-\pi, \pi]$, giving exact answers.

35 $2\sin^2 x - \cos x = 1$ for $x \in [0, 2\pi]$. Find the exact value(s) of x.

36 Solve for x: $\sin x + \cos x = 1$ where $0 \leqslant x \leqslant \pi$.

37 Find x if $\arcsin(2x - 3) = -\frac{\pi}{6}$.

38 If $\cos 2\alpha = \sin^2\alpha$, find the exact value of $\cot\alpha$.

39 Solve the equation $\cot\theta + \tan\theta = 2$ for $\theta \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$.

40 Solve for x where $x \in [-\pi, 3\pi]$, giving exact answers:

a $\sqrt{3}\tan\left(\frac{x}{2}\right) = -1$ **b** $\sqrt{3} + 2\sin(2x) = 0$.

41 Find the exact solutions of $\sin x + \sqrt{3}\cos x = 0$.

42 If $\dfrac{\sin\theta + 2\cos\theta}{\sin\theta - \cos\theta} = 2$, find the exact value of $\tan 2\theta$.

TOPIC 4: MATRICES

A **matrix** is a rectangular array of numbers arranged in **rows** and **columns**.

If a matrix has m rows and n columns, its **order** is $m \times n$.

Two matrices are **equal** if they have the same order and the elements in corresponding positions are equal.

To **add** two matrices of the same order, we **add** the elements in corresponding positions.

To **subtract** two matrices of the same order, we **subtract** the elements in corresponding positions.

To multiply a matrix by a **scalar**, we multiply every element by that scalar.

We can only **multiply** matrix **A** by matrix **B** if the number of columns in **A** equals the number of rows in **B**.

If **A** is $m \times p$ and **B** is $p \times n$ then the product matrix **AB** is $m \times n$. The element in the rth row and cth column of **AB** is the sum of the products of the elements in the rth row of **A** with the corresponding elements in the cth column of **B**.

In general, $\mathbf{AB} \neq \mathbf{BA}$.

A **zero matrix O** is a matrix in which all elements are zero.

$\mathbf{A} + \mathbf{O} = \mathbf{O} + \mathbf{A} = \mathbf{A}$ for all matrices **A** of the same order.

An **identity matrix I** is a square matrix with 1s along the leading diagonal and zeros everywhere else.

$\mathbf{AI} = \mathbf{IA} = \mathbf{A}$ for all square matrices **A** of the same order.

The **determinant** of the matrix $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is det **A** or $|\mathbf{A}| = ad - bc$.

The **inverse** of square matrix **A** is the matrix $\mathbf{A}^{-1}$ such that $\mathbf{AA}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$.

If $|\mathbf{A}| = 0$ then $\mathbf{A}^{-1}$ does not exist and **A** is **singular**.

If $|\mathbf{A}| \neq 0$ then **A** is **invertible**.

If $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $\mathbf{A}^{-1} = \dfrac{1}{|\mathbf{A}|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.