Name: KEY

Need more difficult Sem. exam grading scale.

IB Mathematics HL Year 1

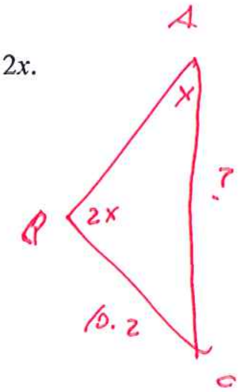
Trigonometry SSA

Calculator Section

35 Minutes

e.g. # mark = +5 to numerator & denominator

1. The triangle ABC has an obtuse angle at B, BC = 10.2, $\hat{A} = x$ and $\hat{B} = 2x$.



(a) Find AC, in terms of cos x.

$$\frac{\sin x}{10.2} = \frac{\sin 2x}{AC}$$

$$\therefore AC = \frac{10.2 \sin 2x}{\sin x} = \frac{(10.2) 2 \cancel{\sin x} \cos x}{\cancel{\sin x}}$$

$$= 20.4 \cos x$$

(3)

(b) Given that the area of triangle ABC is 52.02 cos x, find angle $\hat{C}$.

$$A = \frac{1}{2} ab \sin \theta = 52.02 \cos x$$

$$\frac{1}{2}(10.2)(20.4 \cancel{\cos x}) \sin C = 52.02 \cancel{\cos x}$$

$$\therefore \sin C = \frac{2 \times 52.02}{(10.2)(20.4)}$$

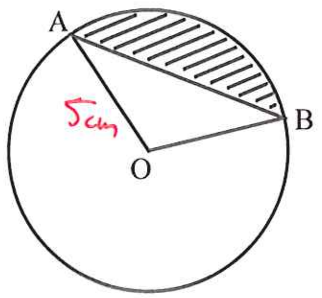
$$= 0.5$$

$$\therefore C = 30°$$

(3)

(Total 6 marks)

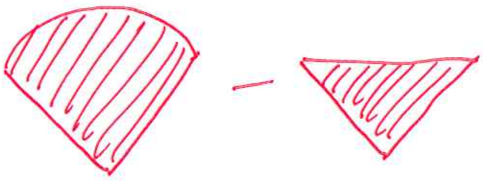
2. The diagram below shows a circle centre O and radius OA = 5 cm. The angle AOB = 135°.



$$135° \rightarrow \times \frac{\pi}{180}$$

$$= \frac{3\pi}{4} \text{ rad.}$$

Find the **exact** area of the shaded region.



$$A = \frac{1}{2}\theta r^2 - \frac{1}{2} ab \sin \theta$$

M2 M2

$$\frac{1}{2}\left(\frac{3\pi}{4}\right)(5)^2 - \frac{1}{2}(5^2) \sin\left(\frac{3\pi}{4}\right)$$

M1

$$A_{Total} = 20.6 \text{ cm}^2$$

A1

$$= \frac{3\pi}{8} \cdot 25 - \frac{1}{2} \cdot 25 \sin\left(\frac{3\pi}{4}\right)$$

$$= \frac{75\pi}{8} - \frac{25}{2}\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{75\pi}{8} - \frac{25\sqrt{2}}{4}$$

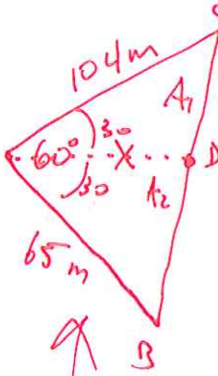
(Total 6 marks)

$$= \frac{75\pi}{8} - \frac{50\sqrt{2}}{8}$$

$$= \frac{75\pi - 50\sqrt{2}}{8} \text{ cm}^2$$

exact

3. A farmer owns a triangular field ABC. The side [AC] is 104 m, the side [AB] is 65 m and the angle between these two sides is 60°.



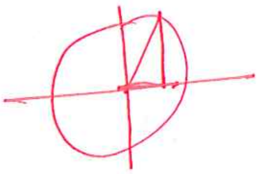
(a) Calculate the length of the third side of the field.

$$a^2 = b^2 + c^2 - 2bc\cos\theta$$
$$= 104^2 + 65^2 - 2(104)(65)\cos 60$$
$$= 8281$$
$$\therefore\ BC = 91\,m$$

(3)

(b) Find the area of the field in the form $p\sqrt{3}$, where p is an integer.

$$A = \tfrac{1}{2}ab\sin\theta$$
$$= \tfrac{1}{2}(104)(65)\sin 60$$
$$= 3380\left(\tfrac{\sqrt{3}}{2}\right) = 1690\sqrt{3}$$



(3)

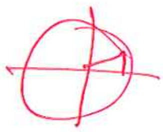
Smaller

Let D be a point on [BC] such that [AD] bisects the 60° angle. The farmer divides the field into two parts by constructing a straight fence [AD] of length x metres.

(c) (i) Show that the area of the smaller part is given by $\frac{65x}{4}$ and find an expression for the area of the larger part.

(4)

Smaller
$$A = \tfrac{1}{2}(65)(x)\sin 30$$
$$= \tfrac{1}{2}65x\left(\tfrac{1}{2}\right)$$
$$= \frac{65x}{4}\ m^2$$



larger
$$A = \tfrac{1}{2}104x\sin 30$$
$$= \tfrac{1}{2}104x\left(\tfrac{1}{2}\right)$$
$$= \frac{104x}{4}$$
$$= 26x\ m^2$$

(ii) Hence, find the value of x in the form $q\sqrt{3}$, where q is an integer.

(8)

$$A_1 + A_2 = A_{Total}$$
$$\frac{65x}{4} + 26x = 1690\sqrt{3}$$
$$\frac{65x}{4} + \frac{104x}{4} = 1690\sqrt{3}$$
$$\frac{169}{4}x = 1690\sqrt{3}$$
$$\Rightarrow x = 40\sqrt{3}$$

(4)

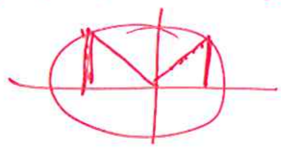
(d) Prove that $\frac{BD}{DC} = \frac{5}{8}$.

(6)
(Total 20 marks)

①
$$\frac{\sin 30}{DC} = \frac{\sin(A\hat{D}C)}{104}$$
$$DC = \frac{104\sin 30}{\sin(A\hat{D}C)}$$

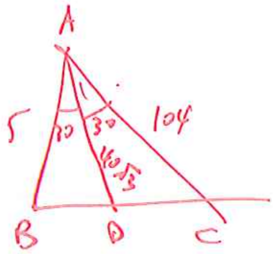
②
$$\frac{\sin 30}{BD} = \frac{\sin(A\hat{D}B)}{65}$$
$$BD = \frac{65\sin 30}{\sin(ADB)}$$

Since
$\angle ADC + \angle ADB = 180$
$\sin(ADC) = \sin(ADB)$



$$\therefore\ \frac{BD}{DC} = \frac{\left(\frac{65\,\cancel{\sin 30}}{\cancel{\sin(\theta)}}\right)}{\left(\frac{104\,\cancel{\sin 30}}{\cancel{\sin\theta}}\right)}$$
$$= \frac{65}{104} = \frac{5}{8} \quad \square$$

→ Jon Armes Method w/ cosine Rule



$$(BD)^2 = 65^2 + (40\sqrt{3})^2 - 2(65)(40\sqrt{3})\cos 30$$
$$= 35$$
$$(DC)^2 = 104^2 + (40\sqrt{3})^2 - 2(104)(40\sqrt{3})\cos 30$$
$$= 56$$

$\cos(30) = \frac{\sqrt{3}}{2}$

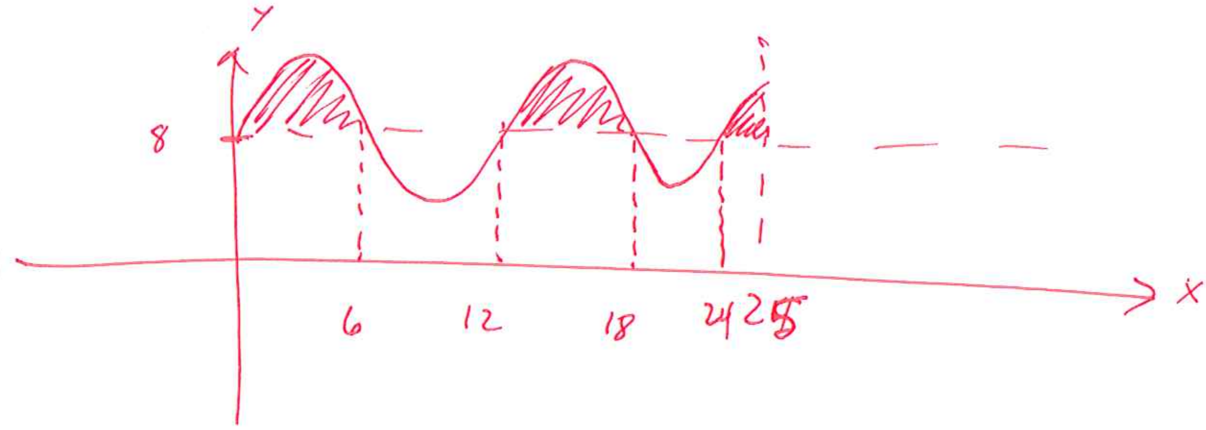
$$\frac{BD}{DC} = \frac{35}{56} = \frac{5}{8} \checkmark$$

4. The depth, $h\ (t)$ metres, of water at the entrance to a harbour at t hours after midnight on a particular day is given by

$$h\ (t) = 8 + 4 \sin\left(\frac{\pi t}{6}\right), 0 \leq t \leq 24.$$

Find the values of t for which $h\ (t) \geq 8$.

(3)



$\therefore\quad 0 \leq t \leq 6$ or

$12 \leq t \leq 18$