

Name:

KEY

IB Mathematics HL Year 1

TRIGONOMETRY SSA

Non-Calculator Section

35 Minutes

1. Given that  $a\sin 4x + b\sin 2x = 0$ , for  $0 < x < \frac{\pi}{2}$ , find an expression for  $\cos^2 x$  in terms of  $a$  and  $b$ .

$$\begin{aligned}
 a\sin(2 \cdot 2x) + b\sin 2x &= 0 \\
 a[2\sin 2x \cos 2x] + b\sin 2x &= 0 \\
 2a\sin 2x \cos 2x + b\sin 2x &= 0 \\
 \sin 2x(2a\cos 2x + b) &= 0 \\
 \therefore \cancel{\sin 2x} = 0 \quad \text{or} \quad 2a\cos 2x + b = 0
 \end{aligned}$$

done:

$$\begin{aligned}
 \cos 2x &= -\frac{b}{2a} \\
 2\cos^2 x - 1 &= -\frac{b}{2a} \\
 2\cos^2 x &= -\frac{b}{2a} + 1
 \end{aligned}$$

double angle.

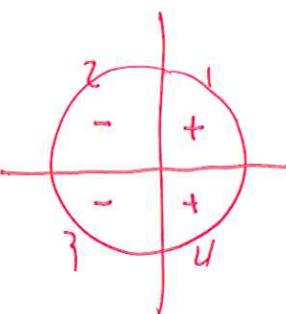
$$\begin{aligned}
 \cos^2 x &= \frac{-b}{4a} + \frac{1}{2} \\
 &= \frac{2a - b}{4a}
 \end{aligned}$$

double angle.

(Total 6 marks)

2. The angle  $\theta$  satisfies the equation  $2\tan^2 \theta - 5\sec \theta - 10 = 0$ , where  $\theta$  is in the second quadrant. Find the exact value of  $\sec \theta$ .

Space

$$\begin{aligned}
 \sec \theta &= \frac{1}{\cos \theta} \quad \therefore \\
 &\quad \begin{aligned}
 &\text{get } \tan^2 \theta \rightarrow \sec \theta \\
 &\therefore 1 + \tan^2 \theta = \sec^2 \theta \\
 &\text{or } \tan^2 \theta = \sec^2 \theta - 1 \\
 &2(\sec^2 \theta - 1) - 5\sec \theta - 10 = 0 \\
 &2\sec^2 \theta - 2 - 5\sec \theta - 10 = 0 \\
 &2\sec^2 \theta - 5\sec \theta - 12 = 0 \\
 &\text{looks like } 2x^2 - 5x - 12 = 0 \quad \downarrow x = \sec \theta \\
 &\therefore (2x + 3)(x - 4) = 0 \\
 &\therefore 2x + 3 = 0 \quad \text{or} \quad x - 4 = 0 \\
 &x = -\frac{3}{2} \quad x = 4
 \end{aligned}
 \end{aligned}$$


$\therefore \sec \theta = -\frac{3}{2}$  or ~~4~~

choose

because  $Q_2$ .

(Total 6 marks)

3. Solve  $2\sin 3x + \sqrt{3} = 0$  for  $0 \leq x \leq 2\pi$ .  $\frac{18\pi}{9}$

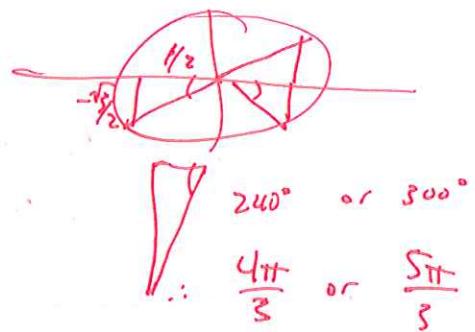
$$2\sin 3x = -\sqrt{3}$$

$$\sin 3x = -\frac{\sqrt{3}}{2}$$

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

$$\therefore 3x = \left\{ \begin{array}{l} \frac{4\pi}{3} \\ \frac{5\pi}{3} \end{array} \right. + 2\pi k$$

$$x = \left\{ \begin{array}{l} \frac{4\pi}{9} \\ \frac{5\pi}{9} \end{array} \right. + \frac{2\pi k}{3} \Rightarrow \frac{6\pi k}{9}$$



$R = \theta$	$k = 1$	$k = 2$	$k = 3$	(Total 6 marks)
$\frac{4\pi}{9}$	$\frac{10\pi}{9}$	$\frac{16\pi}{9}$	X	
$\frac{5\pi}{9}$	$\frac{11\pi}{9}$	$\frac{17\pi}{9}$	X	

4. Prove that  $\frac{\sin 4\theta(1 - \cos 2\theta)}{\cos 2\theta(1 - \cos 4\theta)} = \tan \theta$ , for  $0 < \theta < \frac{\pi}{2}$ , and  $\theta \neq \frac{\pi}{4}$ .

Difficult

$$\begin{aligned}
 & \frac{\sin 2\cdot 2\theta (1 - \cos 2\theta)}{\cos 2\theta (1 - \cos 2\cdot 2\theta)} \\
 &= \frac{2\sin 2\theta \cos 2\theta (1 - \cos 2\theta)}{\cos 2\theta (1 - \cos 2\cdot 2\theta)} \\
 &= \frac{2\sin 2\theta (1 - \cos 2\theta)}{1 - \cos 2\cdot 2\theta} \\
 &\quad \cancel{2\sin 2\theta} \cancel{1 - \cos 2\theta} \cancel{\cos 2\theta} \\
 &= \frac{2\sin 2\theta (1 - (\cos^2 \theta - \sin^2 \theta))}{1 - (\cos^2 2\theta - \sin^2 2\theta)} \\
 &= \frac{2\sin 2\theta ((1 - \cos^2 \theta) + \sin^2 \theta)}{1 - \cos^2 2\theta + \sin^2 2\theta} \\
 &= \frac{2\sin 2\theta (\sin^2 \theta + \sin^2 \theta)}{\sin^2 2\theta + \sin^2 2\theta} \\
 &= \frac{2\sin 2\theta (2\sin^2 \theta)}{2\sin^2 2\theta} \\
 &= \frac{2\sin 2\theta \cdot \sin^2 \theta}{\sin 2\theta \cdot \sin 2\theta} \\
 &= \frac{2\sin^2 \theta}{2\sin \theta \cos \theta} \\
 &= \frac{2\sin \theta}{2\cos \theta} \\
 &= \tan \theta
 \end{aligned}$$

(Total 5 marks)

5

/11

Alternate proof:

(Benjamin)

$$\begin{aligned}& \frac{\sin 4\theta (1 - \cos 2\theta)}{\cos 2\theta (1 - \cos 4\theta)} \\&= \frac{2\sin(2\theta)\cos(2\theta)(1 - \cos 2\theta)}{\cos 2\theta (1 - \cos 4\theta)} \\&= \frac{2\sin(2\theta)(1 - \cos 2\theta)}{1 - (1 - 2\sin^2(2\theta))} \\&= \frac{2\sin(2\theta)(1 - \cos 2\theta)}{2\sin^2(2\theta)} \\&= \frac{1 - \cos 2\theta}{\sin 2\theta} \\&= \frac{1 - (1 - 2\sin^2\theta)}{2\sin\theta\cos\theta} \\&= \frac{1 + 2\sin^2\theta}{2\sin\theta\cos\theta} \\&= \frac{\sin\theta}{\cos\theta} \\&= \tan\theta \quad \square\end{aligned}$$

$\left. \begin{array}{l} \cos 4\theta \\ = \cos 2(2\theta) \\ = 1 - 2\sin^2(2\theta) \end{array} \right\}$

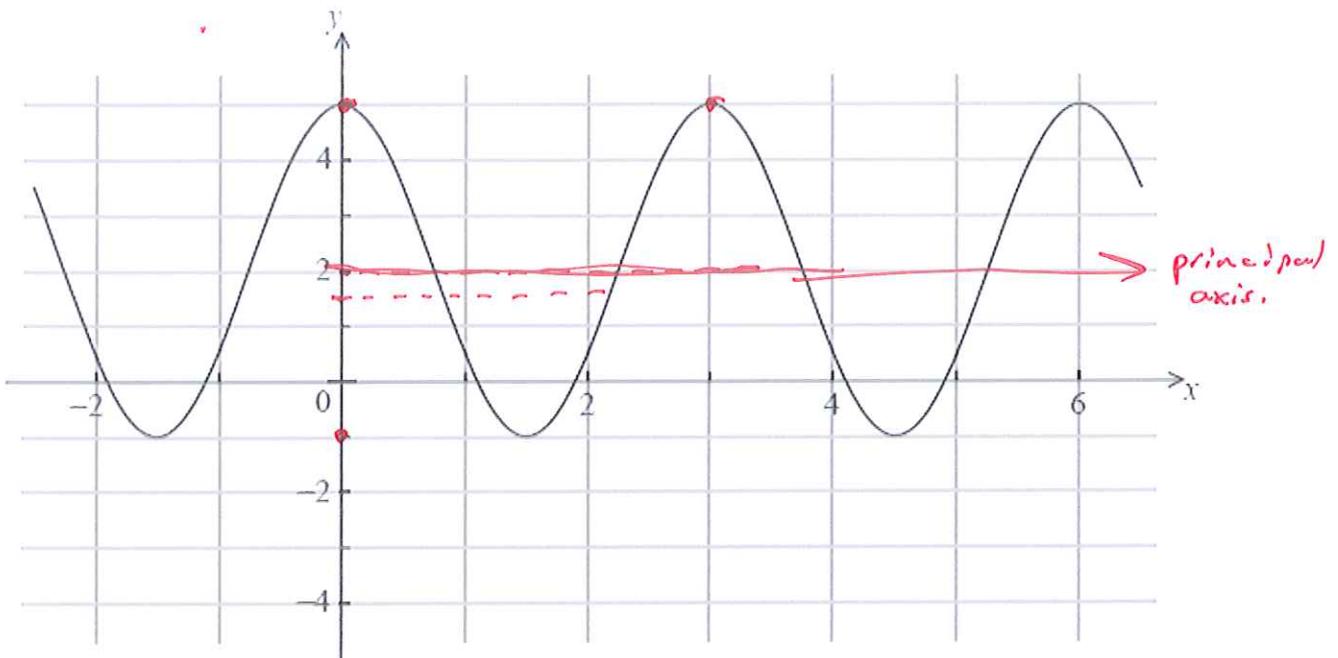
(4) Tonante Method:

$$\begin{aligned}
 & \frac{\sin \theta (1 - \cos 2\theta)}{\cos 2\theta (1 - \cos 4\theta)} \\
 = & \frac{2\sin(2\theta)\cos(\theta)(1-\cos 2\theta)}{\cos(2\theta)(1-\cos 4\theta)} \\
 = & \frac{2[2\sin \theta \cos \theta](1 - \cos 2\theta)}{1 - \cos 4\theta} \\
 = & \frac{4\sin \theta \cos \theta (1 - \cos 2\theta)}{(1 - (1 - 2\sin^2(2\theta)))} \\
 = & \frac{4\sin \theta \cos \theta (1 - (1 - 2\sin^2\theta))}{2\sin^2(2\theta)} \\
 = & \frac{4\sin \theta \cos \theta \cdot 2\sin^2\theta}{2\sin^2(2\theta)} \\
 = & \frac{8\sin^3\theta \cos\theta}{2\sin 2\theta \cdot \sin 2\theta} \\
 = & \frac{8\sin^3\theta \cos\theta}{2(2\sin\theta\cos\theta)(2\sin\theta\cos\theta)} \\
 = & \frac{8\sin^3\theta \cos\theta}{8\sin^2\theta \cos^2\theta} \\
 = & \frac{\sin\theta}{\cos\theta} \\
 = & \tan\theta
 \end{aligned}$$

□

$$\begin{aligned}
 \cos 4\theta &= \cos 2(2\theta) \\
 &= 1 - 2\sin^2(2\theta)
 \end{aligned}$$

5. The graph below shows  $y = a \cos(bx) + c$ .



Find the value of  $a$ , the value of  $b$  and the value of  $c$ .

(Total 4 marks)

$$\therefore \cancel{A = 3} \quad \therefore c = 2 \\ A = 3 \\ P = \frac{2\pi}{b} \\ 3 = \frac{2\pi}{b} \quad \checkmark \\ \therefore b = \frac{2\pi}{3}$$

EXTRA CREDIT: Attempt ONLY if you are finished with the rest of the test.

**EC** (a) Show that  $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$ .

$$\frac{2\sin \theta \cos \theta}{1 + (2\cos^2 \theta - 1)} \\ = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

(2)

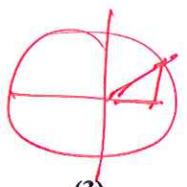
(b) Hence find the value of  $\cot \frac{\pi}{8}$  in the form  $a + b\sqrt{2}$ , where  $a, b \in \mathbb{Z}$ .

$$\therefore \tan \frac{\pi}{8} = \frac{\sin \frac{\pi}{4}}{1 + \cos \frac{\pi}{4}}$$

$$\therefore \cot \left( \frac{\pi}{8} \right) = \frac{1 + \cos \frac{\pi}{4}}{\sin \frac{\pi}{4}} \\ = \frac{1 + \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} \\ = (1 + \frac{1}{\sqrt{2}})\sqrt{2}$$

$$= \sqrt{2} + 1 \\ = 1 + \sqrt{2}$$

(Total 5 marks)



(3)