General

The new course for mathematics SL will be examined for the first time in May 2006.

This publication supplements and should be read alongside the *Mathematics SL guide*, published in April 2004, which contains the curriculum and assessment requirements for the whole course. Where appropriate, extracts from the guide have been reproduced here for ease of reference. This publication offers useful suggestions and guidance for the implementation of the internally assessed component—the portfolio. General regulations and procedures relating to internal assessment have not been reproduced here but can be found in the relevant section of the *Vade Mecum*.

Every student must produce a portfolio containing **two** pieces of work completed during the course. Each piece of work in the portfolio is internally assessed by the teacher against criteria that are related to the objectives of the mathematics SL course. A sample of student portfolios from each school is then externally moderated to ensure uniformity of standards. The portfolio is worth 20% of the total score for the mathematics SL course.

Each task in a portfolio is assigned by the teacher. The tasks must be based on different areas of the course and represent **two** types of tasks: mathematical investigation (type I) and mathematical modelling (type II). The definitions of the different types of tasks are given on pages 2 and 3. Students must submit a portfolio containing **two** pieces of work, and it is recommended that teachers set more than two tasks.

This publication contains support material that exemplifies standards of assessment in the portfolio together with specimen tasks and student work, as well as guidance on the introduction and management of portfolio work in the classroom. This publication contains material contributed by teachers to help other teachers, across all aspects of portfolio work. Teachers are reminded that tasks that were written for the previous course do not lend themselves to achieving the highest levels in some criteria.

The IBO welcomes comments from teachers on this publication and also any ideas for teaching the course that may be of value to other teachers in Diploma Programme schools. Comments should be addressed to the group 5 subject area manager at the IBO.

The purpose of the portfolio

The purpose of the portfolio is to provide students with opportunities to be rewarded for mathematics carried out under ordinary conditions, that is, without the time limitations and pressure associated with written examinations. Consequently, the emphasis should be on good mathematical writing and thoughtful reflection.

The portfolio is also intended to provide students with opportunities to increase their understanding of mathematical concepts and processes. It is hoped that, by doing portfolio work, students benefit from these mathematical activities and find them both stimulating and rewarding.

The specific purposes of portfolio work are to:

- develop students' personal insight into the nature of mathematics and to develop their ability to ask their own questions about mathematics
- provide opportunities for students to complete extended pieces of mathematical work without the time constraints of an examination

- enable students to develop individual skills and techniques, and to allow them to experience the satisfaction of applying mathematical processes on their own
- provide students with the opportunity to experience for themselves the beauty, power and usefulness of mathematics
- provide students with the opportunity to discover, use and appreciate the power of a calculator or computer as a tool for doing mathematics
- enable students to develop the qualities of patience and persistence, and to reflect on the significance of the results they obtain
- provide opportunities for students to show, with confidence, what they know and what they can do.

Objectives

The portfolio is internally assessed by the teacher and externally moderated by the IBO. Assessment criteria have been developed to relate to the objectives of the mathematics SL course. In developing these criteria, particular attention has been given to the objectives listed here, since these cannot be easily addressed by means of timed, written examinations.

Where appropriate in the portfolio, students are expected to:

- know and use appropriate notation and terminology
- organize and present information and data in tabular, graphical and/or diagrammatic forms
- recognize patterns and structures in a variety of situations, and make generalizations
- demonstrate an understanding of and the appropriate use of mathematical modelling
- recognize and demonstrate an understanding of the practical applications of mathematics
- use appropriate technological devices as mathematical tools.

Tasks

Type I—mathematical investigation

While many teachers incorporate a problem-solving approach into their classroom practice, students also should be given the opportunity formally to carry out investigative work. The mathematical investigation is intended to highlight that:

- the idea of investigation is fundamental to the study of mathematics
- investigation work often leads to an appreciation of how mathematics can be applied to solve problems in a broad range of fields
- the discovery aspect of investigation work deepens understanding and provides intrinsic motivation
- during the process of investigation, students acquire mathematical knowledge, problem-solving techniques, a knowledge of fundamental concepts and an increase in self-confidence.

All investigations develop from an initial problem, the starting point. The problem must be clearly stated and contain no ambiguity. In addition, the problem should:

- provide a challenge and the opportunity for creativity
- contain multi-solution paths, that is, contain the potential for students to choose different courses of action from a range of options.

Essential skills to be assessed

- Producing a strategy
- Generating data
- Recognizing patterns or structures
- Searching for further cases
- Forming a general statement
- Testing a general statement
- Justifying a general statement
- Appropriate use of technology

Type II—mathematical modelling

Problem solving usually elicits a process-oriented approach, whereas mathematical modelling requires an experimental approach. By considering different alternatives, students can use modelling to arrive at a specific conclusion, from which the problem can be solved. To focus on the actual process of modelling, the assessment should concentrate on the appropriateness of the model selected in relation to the given situation, and on a critical interpretation of the results of the model in the real-world situation chosen.

Mathematical modelling involves the following skills.

- Translating the real-world problem into mathematics
- Constructing a model
- Solving the problem
- Interpreting the solution in the real-world situation (that is, by the modification or amplification of the problem)
- Recognizing that different models may be used to solve the same problem
- Comparing different models
- Identifying ranges of validity of the models
- Identifying the possible limits of technology
- Manipulating data

Essential skills to be assessed

- Identifying the problem variables
- Constructing relationships between these variables
- Manipulating data relevant to the problem
- Estimating the values of parameters within the model that cannot be measured or calculated from the data
- Evaluating the usefulness of the model
- Communicating the entire process
- Appropriate use of technology

Introduction

Please see pages 2 and 3 for definitions of the type I (mathematical investigation) and type II (mathematical modelling) tasks. Each piece of work in the portfolio should be assessed against the following six criteria.

Type I—mathematical investigation	Type II—mathematical modelling
A: Use of notation and terminology	A: Use of notation and terminology
B: Communication	B: Communication
C: Mathematical process—searching for patterns	C: Mathematical process—developing a model
D: Results—generalization	D: Results—interpretation
E: Use of technology	E: Use of technology
F: Quality of work	F: Quality of work

Descriptions of the achievement levels for each of these six assessment criteria appear in the *Mathematics SL guide* and are reproduced in this section for ease of reference. Instructions for applying the criteria are also included in the guide. Please note that criteria C and D are different for each type of task. It is particularly important to note that each achievement level represents the **minimum** requirement for that level to be awarded.

The final mark

Each portfolio must contain two pieces of work (if more than two pieces have been completed the best two should be selected for submission). The final mark for each portfolio is obtained by adding the achievement levels for both tasks together to give a total out of 40. For example:

	A		B		C		D		E		F		
Type I	1		3		3		2		3		2		
Type II	2		3		4		2		2		2		
	3	+	6	+	7	+	4	+	5	+	4	=	29

The final mark is 29.

Incomplete portfolios

If only one piece of work is submitted, award zero for each criterion for the missing work. For example:

	A		B		C		D		E		F		
Type I	1		3		3		2		3		2		
Type II	0		0		0		0		0		0		
	1	+	3	+	3	+	2	+	3	+	2	=	14

The final mark is 14.

Achievement levels

Criterion A: Use of notation and terminology

Achievement level	Descriptor
0	The student does **not** use appropriate notation and terminology.
1	The student uses **some** appropriate notation and/or terminology.
2	The student uses appropriate notation and terminology in a **consistent** manner and does so throughout the work.

Tasks will probably be set before students are aware of the notation and/or terminology to be used. Therefore the key idea behind this criterion is to assess how well students' use of terminology describes the context. Teachers should provide an appropriate level of background knowledge in the form of notes given to students at the time the task is set.

Correct mathematical notation is required, but it can be accompanied by calculator notation, particularly when students are substantiating their use of technology.

This criterion addresses appropriate use of mathematical symbols (for example, use of " $\approx$ " instead of " $=$ " and proper vector notation).

Word processing a document does not increase the level of achievement for this criterion or for criterion B.

Students should take care to write in appropriate mathematical symbols if the word-processing software does not supply them. For example, using $x \wedge 2$ instead of x^2 would be considered a lack of proper usage and the student would not achieve level 2.

Criterion B: Communication

Achievement level	Descriptor
0	The student **neither provides** explanations **nor uses** appropriate forms of representation (for example, symbols, tables, graphs and/or diagrams).
1	The student **attempts** to provide explanations or uses **some** appropriate forms of representation (for example, symbols, tables, graphs and/or diagrams).
2	The student provides **adequate** explanations or arguments, and communicates them using appropriate forms of representation (for example, symbols, tables, graphs and/or diagrams).
3	The student provides **complete, coherent** explanations or arguments, and communicates them clearly using appropriate forms of representation (for example, symbols, tables, graphs and/or diagrams).

This criterion also assesses how coherent the work is. The work can achieve a good mark if the reader does not need to refer to the wording used to set the task. In other words, the task can be marked independently.

Level 2 cannot be achieved if the student only writes down mathematical computations without explanation.

Graphs, tables and diagrams should accompany the work in the appropriate place and not be attached to the end of the document. Graphs must be correctly labelled and must be neatly drawn on graph paper. Graphs generated by a computer program or a calculator "screen dump" are acceptable provided that all items are correctly labelled, even if the labels are written in by hand. Colour keying the graphs can increase clarity of communication.

Criterion C: Mathematical process

Type I—mathematical investigation: searching for patterns

Achievement level	Descriptor
0	The student **does not attempt** to use a mathematical strategy.
1	The student **uses** a mathematical strategy to produce data.
2	The student **organizes** the data generated.
3	The student **attempts to analyse** data to enable the formulation of a general statement.
4	The student **successfully analyses** the **correct** data to enable the formulation of a general statement.
5	The student **tests** the validity of the general statement by considering further examples.

Students can only achieve level 3 if the amount of data generated is sufficient to warrant an analysis.

Type II—mathematical modelling: developing a model

Achievement level	Descriptor
0	The student **does not define** variables, parameters or constraints of the task.
1	The student defines **some** variables, parameters or constraints of the task.
2	The student defines variables, parameters **and** constraints of the task **and** attempts to create a mathematical model.
3	The student **correctly analyses** variables, parameters and constraints of the task to enable the **formulation** of a mathematical model that is **relevant** to the task and consistent with the level of the course.
4	The student **considers** how well the model fits the data.
5	The student **applies** the model to other situations.

At achievement level 5, applying the model to other situations could include, for example, a change of parameter or more data.

Criterion D: Results

Type I—mathematical investigation: generalization

Achievement level	Descriptor
0	The student **does not produce** any general statement consistent with the patterns and/or structures generated.
1	The student **attempts** to produce a general statement that is consistent with the patterns and/or structures generated.
2	The student **correctly** produces a general statement that is consistent with the patterns and/or structures generated.
3	The student **expresses** the **correct** general statement in **appropriate mathematical terminology**.
4	The student **correctly states** the scope or limitations of the general statement.
5	The student gives a **correct** informal justification of the general statement.

A student who gives a correct formal proof of the general statement that does not take into account scope or limitations would achieve level 4.

Type II—mathematical modelling: interpretation

Achievement level	Descriptor
0	The student **has not arrived** at any results.
1	The student has arrived at **some** results.
2	The student **has not interpreted** the reasonableness of the results of the model in the **context of the task**.
3	The student has **attempted** to interpret the reasonableness of the results of the model in the **context of the task**, to the appropriate degree of accuracy.
4	The student has **correctly** interpreted the reasonableness of the results of the model in the context of the task, to the appropriate degree of accuracy.
5	The student has **correctly** and **critically** interpreted the reasonableness of the results of the model in the context of the task, **including** possible limitations and modifications of these results, to the appropriate degree of accuracy.

Criterion E: Use of technology

Achievement level	Descriptor
0	The student uses a calculator or computer for **only routine** calculations.
1	The student **attempts** to use a calculator or computer in a manner that could enhance the development of the task.
2	The student makes **limited** use of a calculator or computer in a manner that enhances the development of the task.
3	The student makes **full** and **resourceful** use of a calculator or computer in a manner that **significantly** enhances the development of the task.

The level of calculator or computer technology varies from school to school. Therefore teachers should state the level of the technology that is available to their students.

Using a computer and/or a graphic display calculator (GDC) to generate only graphs or tables may not significantly contribute to the development of the task.

Criterion F: Quality of work

Achievement level	Descriptor
0	The student has shown a **poor** quality of work.
1	The student has shown a **satisfactory** quality of work.
2	The student has shown an **outstanding** quality of work.

Students who satisfy all the requirements correctly achieve level 1. For a student to achieve level 2, work must show precision, insight and a sophisticated level of mathematical understanding.

Overview of assessment criteria for type I tasks

	Criterion A: Use of notation and terminology	Criterion B: Communication	Criterion C: Mathematical process—searching for patterns	Criterion D: Results—generalization	Criterion E: Use of technology	Criterion F: Quality of work
0	The student does **not** use appropriate notation and terminology.	The student **neither provides** explanations **nor uses** appropriate forms of representation (for example, symbols, tables, graphs and/or diagrams).	The student **does not attempt** to use a mathematical strategy.	The student **does not produce** any general statement consistent with the patterns and/or structures generated.	The student uses a calculator or computer for **only routine** calculations.	The student has shown a **poor** quality of work.
1	The student uses **some** appropriate notation and/or terminology.	The student **attempts** to provide explanations or uses **some** appropriate forms of representation (for example, symbols, tables, graphs and/or diagrams).	The student **uses** a mathematical strategy to produce data.	The student **attempts** to produce a general statement that is consistent with the patterns and/or structures generated.	The student **attempts** to use a calculator or computer in a manner that could enhance the development of the task.	The student has shown a **satisfactory** quality of work.
2	The student uses appropriate notation and terminology in a **consistent** manner and does so throughout the work.	The student provides **adequate** explanations or arguments, and communicates them using appropriate forms of representation (for example, symbols, tables, graphs and/or diagrams).	The student **organizes** the data generated.	The student **correctly** produces a general statement that is consistent with the patterns and/or structures generated.	The student makes **limited** use of a calculator or computer in a manner that enhances the development of the task.	The student has shown an **outstanding** quality of work.
3		The student provides **complete, coherent** explanations or arguments, and communicates them clearly using appropriate forms of representation (for example, symbols, tables, graphs and/or diagrams).	The student **attempts to analyse** data to enable the formulation of a general statement.	The student **expresses** the **correct** general statement in **appropriate mathematical terminology**.	The student makes **full** and **resourceful** use of a calculator or computer in a manner that **significantly** enhances the development of the task.	
4			The student **successfully analyses** the **correct** data to enable the formulation of a general statement.	The student **correctly states** the scope or limitations of the general statement.		
5			The student **tests** the validity of the general statement by considering further examples.	The student gives a **correct**, informal justification of the general statement.		