Vectors - page 1 of 2

1) The diagram shows a trapezium *PQRS*.
$\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{QR} = \mathbf{b}$.
PS is three times the length of *QR*.

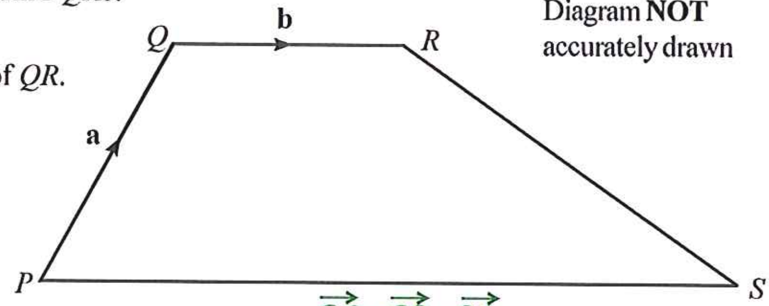

Diagram **NOT** accurately drawn

$\overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{PS}$
$= -a + 3b$

Find, in terms of **a** and **b**, expressions for

a) $\overrightarrow{QP} = -a$ b) $\overrightarrow{PR} = a + b$ c) $\overrightarrow{PS} = 3b$ d) $\overrightarrow{QS} = 3b - a$

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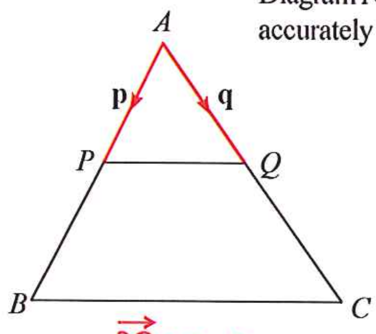
2) In triangle *ABC*, *P* and *Q* are the midpoints of *AB* and *AC*.

$\overrightarrow{AP} = \mathbf{p}$ and $\overrightarrow{AQ} = \mathbf{q}$.

Diagram **NOT** accurately drawn



a) Find, in terms of **p** and **q**, expressions for

(i) $\overrightarrow{PQ}$ $q - p$ (ii) $\overrightarrow{AB}$ $2p$ (iii) $\overrightarrow{AC}$ $2q$ (iv) $\overrightarrow{BC}$ $2q - 2p$

b) Use your results from (a) to prove that *PQ* is parallel to *BC*.

$\overrightarrow{PQ} = q - p$
$\overrightarrow{BC} = 2q - 2p$
$= 2(q - p)$
Therefore $\overrightarrow{PQ}$ is parallel to $\overrightarrow{BC}$

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3)

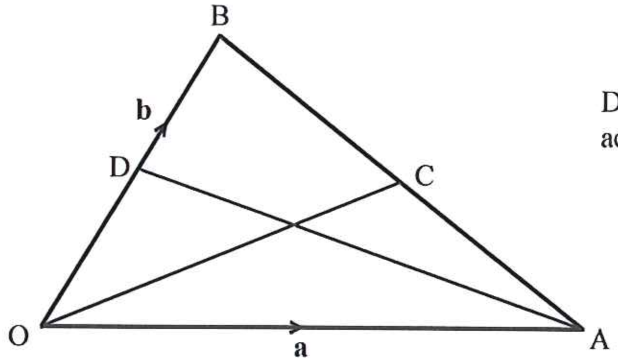

Diagram **NOT** accurately drawn

OAB is a triangle.
D is the midpoint of OB.
C is the midpoint of AB.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$
$\overrightarrow{AC} = \frac{1}{2}\overrightarrow{AB}$
$\overrightarrow{AB} = -a + b$
$\overrightarrow{OC} = a + \frac{1}{2}(-a + b)$
$\overrightarrow{OC} = \frac{1}{2}(a + b)$

(i) Find $\overrightarrow{OC}$ in terms of **a** and **b**.
$\overrightarrow{OC} = \frac{1}{2}(a + b)$

(ii) Show that DC is parallel to OA.

$\overrightarrow{DC} = \overrightarrow{DO} + \overrightarrow{OC}$
$= -\frac{1}{2}b + \frac{1}{2}(a + b)$
$= \frac{1}{2}a$
$\overrightarrow{OA} = a$ Therefore $\overrightarrow{DC}$ is parallel to $\overrightarrow{OA}$

Vectors - page 2 of 2

1)

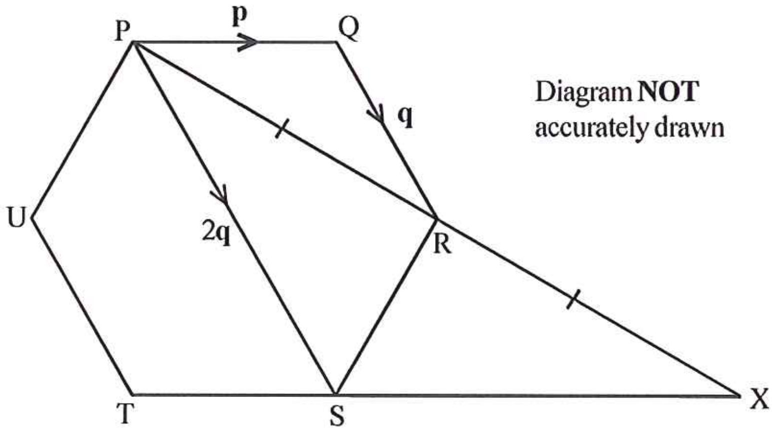

Diagram **NOT** accurately drawn

PQRSTU is a regular hexagon.

$\overrightarrow{PQ} = \mathbf{p}$ $\overrightarrow{QR} = \mathbf{q}$ $\overrightarrow{PS} = 2\mathbf{q}$

a) Find the vector PR in terms of **p** and **q**.

PR = p + q

$\overrightarrow{PR} = \overrightarrow{RX}$

b) Prove that PQ is parallel to SX

$\overrightarrow{SX} = \overrightarrow{SP} + \overrightarrow{PX}$
$= \overrightarrow{SP} + 2\overrightarrow{PR}$
$= -2q + 2(p + q)$
$= -2q + 2p + 2q$
$= 2p$
$\overrightarrow{PQ} = p$ Therefore $\overrightarrow{PQ}$ is parallel to $\overrightarrow{SX}$

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2)

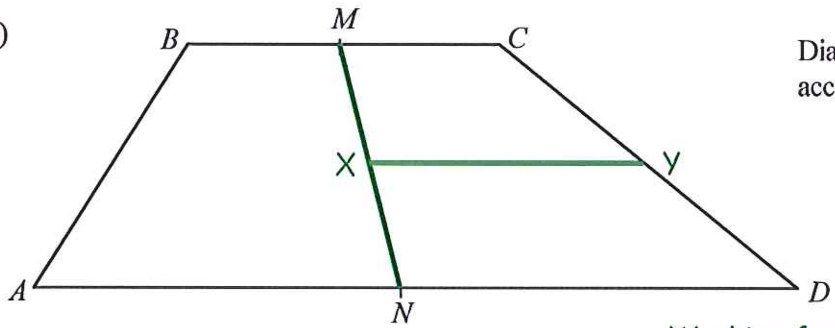

Diagram **NOT** accurately drawn

ABCD is a trapezium with *BC* parallel to *AD*.

$\overrightarrow{AB} = 3\mathbf{b}$ $\overrightarrow{BC} = 3\mathbf{a}$ $\overrightarrow{AD} = 9\mathbf{a}$

M is the midpoint of *BC* and N is the midpoint of *AD*.

a) Find the vector *MN* in terms of **a** and **b**.

$\overrightarrow{MN} = 3a - 3b$

X is the midpoint of *MN* and *Y* is the midpoint of *CD*.

b) Prove that *XY* is parallel to *AD*.

Working for part b)

$\overrightarrow{XY} = \overrightarrow{XN} + \overrightarrow{ND} + \overrightarrow{DY}$
$= \frac{1}{2}\overrightarrow{MN} + \overrightarrow{ND} + \overrightarrow{DY}$
$= \frac{1}{2}(3a - 3b) + 4\frac{1}{2}a + \overrightarrow{DY}$
$= 6a - 1\frac{1}{2}b + \overrightarrow{DY}$
$\overrightarrow{DY} = \frac{1}{2}\overrightarrow{DC}$
$= \frac{1}{2}(\overrightarrow{DA} + \overrightarrow{AB} + \overrightarrow{BC})$
$= -4\frac{1}{2}a + 1\frac{1}{2}b + 1\frac{1}{2}a$
$= 1\frac{1}{2}b - 3a$
$\overrightarrow{XY} = 6a - 1\frac{1}{2}b + 1\frac{1}{2}b - 3a$
$= 3a$
Therefore $\overrightarrow{XY}$ is parallel to $\overrightarrow{AD}$