

Chapter 10

THE UNIT CIRCLE AND RADIAN MEASURE

EXERCISE 10A

1 a $180^\circ = \pi$ radians
 $\therefore 90^\circ = \frac{\pi}{2}$ radians

d $180^\circ = \pi$ radians
 $\therefore 18^\circ = \frac{\pi}{10}$ radians

g $180^\circ = \pi$ radians
 $\therefore 45^\circ = \frac{\pi}{4}$ radians
 $\therefore 225^\circ = \frac{5\pi}{4}$ radians

j $720^\circ = 4 \times 180^\circ$
 $= 4\pi$ radians

m $180^\circ = \pi$ radians
 $\therefore 36^\circ = \frac{\pi}{5}$ radians

2 a 36.7°
 $= 36.7 \times \frac{\pi}{180}$ radians
 ≈ 0.641 radians

d 219.6°
 $= 219.6 \times \frac{\pi}{180}$ radians
 ≈ 3.83 radians

3 a $\frac{\pi}{5}$
 $= \frac{180^\circ}{5}$
 $\therefore 36^\circ$

f $\frac{7\pi}{9}$
 $= \frac{7 \times 180^\circ}{9}$
 $= 140^\circ$

4 a 2°
 $= 2 \times \frac{180}{\pi}$ degrees
 $\approx 114.59^\circ$

d 3.179°
 $= 3.179 \times \frac{180}{\pi}$ degrees
 $\approx 182.14^\circ$

5 a $Degrees$

Degrees	0	45	90	135	180	225	270	315	360
Radians	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π

b $180^\circ = \pi$ radians
 $\therefore 60^\circ = \frac{\pi}{3}$ radians

e $180^\circ = \pi$ radians
 $\therefore 9^\circ = \frac{\pi}{20}$ radians

h $180^\circ = \pi$ radians
 $\therefore 90^\circ = \frac{\pi}{2}$ radians
 $\therefore 270^\circ = \frac{3\pi}{2}$ radians

k $180^\circ = \pi$ radians
 $\therefore 45^\circ = \frac{\pi}{4}$ radians
 $\therefore 315^\circ = \frac{7\pi}{4}$ radians

n $180^\circ = \pi$ radians
 $\therefore 10^\circ = \frac{\pi}{18}$ radians
 $\therefore 80^\circ = \frac{8\pi}{18}$ radians
 $= \frac{4\pi}{9}$ radians

b 137.2°
 $= 137.2 \times \frac{\pi}{180}$ radians
 ≈ 2.39 radians

e 396.7°
 $= 396.7 \times \frac{\pi}{180}$ radians
 ≈ 6.92 radians

c 317.9°
 $= 317.9 \times \frac{\pi}{180}$ radians
 ≈ 5.55 radians

o $180^\circ = \pi$ radians
 $\therefore 10^\circ = \frac{\pi}{18}$ radians
 $\therefore 230^\circ = \frac{23\pi}{18}$ radians

i 317.9°
 $= 317.9 \times \frac{\pi}{180}$ radians
 ≈ 5.55 radians

d 317.9°
 $= 317.9 \times \frac{\pi}{180}$ radians
 ≈ 5.55 radians

e 317.9°
 $= 317.9 \times \frac{\pi}{180}$ radians
 ≈ 5.55 radians

f 317.9°
 $= 317.9 \times \frac{\pi}{180}$ radians
 ≈ 5.55 radians

g 5.267°
 $= 5.267 \times \frac{180}{\pi}$ degrees
 $\approx 49.68^\circ$

h 0.867°
 $= 0.867 \times \frac{180}{\pi}$ degrees
 $\approx 49.68^\circ$

i 0.867°
 $= 0.867 \times \frac{180}{\pi}$ degrees
 $\approx 49.68^\circ$

j 27°
 $= 27 \times \frac{\pi}{180}$ radians
 $= 27^\circ$

k 150°
 $= 150 \times \frac{\pi}{180}$ radians
 $= 150^\circ$

l 22.5°
 $= 22.5 \times \frac{\pi}{180}$ radians
 $= 22.5^\circ$

b	Degrees	0	30	60	90	120	150	180	210	240	270	300	330	360
	Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π

EXERCISE 10B

1 a i arc length $= \left(\frac{41.6}{360}\right) \times 2\pi \times 9$
 ≈ 6.53 cm

b i arc length $= \left(\frac{122}{360}\right) \times 2\pi \times 4.93$
 ≈ 10.5 cm

2 a $\theta = 107.9^\circ$, $l = 5.92$

$$\therefore \left(\frac{107.9}{360}\right) \times 2\pi \times r = 5.92$$

$$\therefore r = \frac{5.92 \times 360}{107.9 \times 2 \times \pi}$$

$$\therefore r \approx 3.14$$
 m

3 a area $= \left(\frac{\theta}{360}\right) \times \pi r^2$
 $\therefore 20.8 = \left(\frac{68.2}{360}\right) \times \pi r^2$

$$\therefore \frac{20.8 \times 360}{68.2 \times \pi} = r^2$$

$$\therefore r = \sqrt{\frac{20.8 \times 360}{68.2 \times \pi}}$$

$$\therefore r \approx 5.91$$
 cm

4 a $l = \left(\frac{\theta}{360}\right) \times 2\pi \times r$
 $\therefore 2.95 = \left(\frac{\theta}{360}\right) \times 2\pi \times 4.3$

$$\therefore \frac{2.95 \times 360}{2 \times \pi \times 4.3} = \theta$$

$$\therefore \theta \approx 39.3^\circ$$

5 a $l = \theta r$
 $\therefore 6 = \theta \times 8$

$$\therefore \theta = \frac{6}{8}$$

$$\therefore \theta = 0.75^\circ$$

b $l = \theta r$
 $\therefore 8.4 = \theta \times 5$

$$\therefore \theta = \frac{8.4}{5}$$

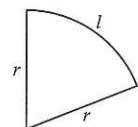
$$\therefore \theta = 1.68^\circ$$

c $l = \phi r$
 $\therefore 31.7 = \phi \times 8$

$$\therefore \phi = \frac{31.7}{8}$$

$$\therefore \phi \approx 3.96^\circ$$

But $\theta = 2\pi - \phi$
 $\therefore \theta \approx 2.32^\circ$



b area $= \left(\frac{\theta}{360}\right) \times \pi r^2$
 $\therefore 30 = \left(\frac{\theta}{360}\right) \times \pi \times 10^2$

$$\therefore \frac{30 \times 360}{\pi \times 100} = \theta$$

$$\therefore \theta \approx 34.4^\circ$$

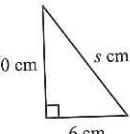
area $= \frac{1}{2} \theta r^2$
 $= \frac{1}{2} (0.75) \times 8^2$
 $= 24$ cm²

area $= \frac{1}{2} \theta r^2$
 $= \frac{1}{2} (1.68) \times 5^2$
 $= 21$ cm²

area $= \frac{1}{2} \phi r^2$
 $= \frac{1}{2} \times \frac{31.7}{8} \times 8^2$
 $= 126.8$ cm²

6 arc length = $r\theta$
 $= \frac{1}{2}r^2\theta$
 $= 5 \times 2$
 $= 10 \text{ cm}$
 $\therefore \text{area} = \frac{1}{2} \times 5^2 \times 2$
 $= 25 \text{ cm}^2$

7 arc length = $r\theta$
 $\therefore 13 = 10\theta$
 $\therefore \theta = \frac{13}{10}$
 $\therefore \text{area} = \frac{1}{2}r^2\theta$
 $= \frac{1}{2} \times 10^2 \times 1.3$
 $= 65 \text{ cm}^2$

8 a 
 $s^2 = 6^2 + 10^2$ {Pythagoras}
 $\therefore s = \sqrt{6^2 + 10^2}$
 $\therefore s \approx 11.6619$
 $\therefore s \approx 11.7$

d arc length = $\left(\frac{\theta}{360}\right) \times 2\pi r$
 $\therefore 37.6991 \approx \frac{\theta}{360} \times 2 \times \pi \times 11.6619$
 $\therefore \theta \approx \frac{37.6991 \times 360}{2 \times \pi \times 11.6619}$
 $\therefore \theta \approx 185^\circ$

b $r = s \approx 11.7$
c arc length = $2\pi \times 6 \approx 37.6991 \approx 37.7 \text{ cm}$

9 a $\tan \alpha = \frac{5}{15}$
 $\therefore \alpha = \tan^{-1}\left(\frac{1}{3}\right)$
 $\therefore \alpha \approx 18.43^\circ$

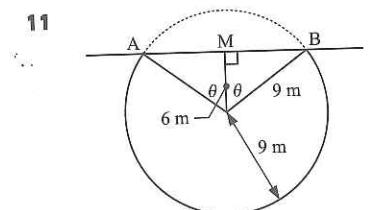
b $\theta + 2\alpha = 180^\circ$ {angles on a line}
 $\therefore \theta \approx 180 - 2 \times 18.43$
 $\therefore \theta \approx 143.1^\circ$

Note: $r^2 = 5^2 + 15^2$
 $\therefore r^2 = 250$

c area = $2 \times \text{area of } \triangle CDB + \text{area of sector}$
 $\approx 2 \times \frac{1}{2} \times 15 \times 5 + \left(\frac{143.1}{360}\right) \times \pi \times 250$
 $\approx 387 \text{ m}^2$

10 a $l = \left(\frac{\theta}{360}\right) \times 2\pi r$
 $= \frac{\frac{1}{60}}{360} \times 2 \times \pi \times 6370 \text{ km}$
 $\approx 1.852957 \text{ km}$
 $\approx 1.853 \text{ km}$

b speed = $\frac{\text{distance}}{\text{time}}$ \therefore time = $\frac{\text{distance}}{\text{speed}}$
 $= \frac{2130 \text{ km}}{480 \text{ n miles h}^{-1}}$
 $= \frac{2130 \text{ km}}{480 \times 1.853 \text{ km h}^{-1}}$
 $\approx 2.3947 \text{ hours}$
 $\approx 2 \text{ hours } 24 \text{ min}$

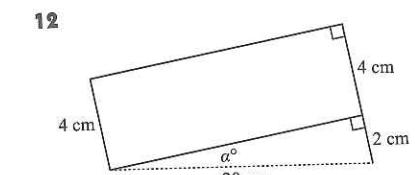


$\cos \theta = \frac{6}{9} = \frac{2}{3}$
 $\therefore \theta = \cos^{-1}\left(\frac{2}{3}\right)$
 $\therefore \theta \approx 48.19^\circ$

So, $360 - 2\theta \approx 263.62^\circ$

Now $MB = \sqrt{9^2 - 6^2} = \sqrt{45}$

available feeding area
= area of \triangle + area of sector
 $= \frac{1}{2} \times 2 \times \sqrt{45} \times 6$
 $+ \left(\frac{263.62}{360}\right) \times \pi \times 9^2$
 $\approx 227 \text{ m}^2$



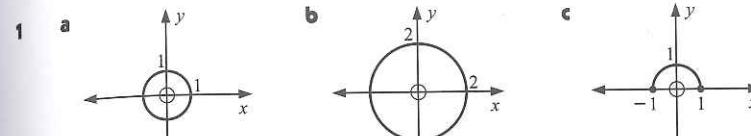
a $\sin \alpha = \frac{2}{20} = 0.1$
 $\therefore \alpha = \sin^{-1}(0.1)$
 $\therefore \alpha \approx 5.739^\circ$

c $\phi + \theta = 360^\circ$
 $\therefore \phi \approx 360 - 168.5$
 $\therefore \phi \approx 191.5^\circ$

b $\theta + 90 + 90 + 2\alpha = 360^\circ$
 $\therefore \theta = 180 - 2\alpha$
 $\approx 180 - 2 \times 5.739$
 $\approx 168.5^\circ$

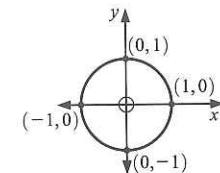
d length of belt
 $= 2 \times \sqrt{20^2 - 2^2}$
 $+ \frac{\theta}{360} \times 2\pi \times 4$
 $+ \frac{\phi}{360} \times 2\pi \times 6$
 $\approx 71.62 \text{ cm}$

EXERCISE 10C.1

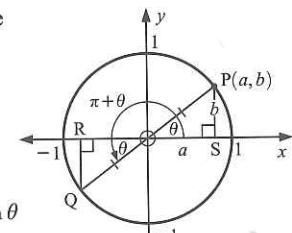


- 2 a i $A(\cos 26^\circ, \sin 26^\circ)$, $B(\cos 146^\circ, \sin 146^\circ)$, $C(\cos 199^\circ, \sin 199^\circ)$
ii $A(0.899, 0.438)$, $B(-0.829, 0.559)$, $C(-0.946, -0.326)$
- b i $A(\cos 123^\circ, \sin 123^\circ)$, $B(\cos 251^\circ, \sin 251^\circ)$, $C(\cos(-35^\circ), \sin(-35^\circ))$
ii $A(-0.545, 0.839)$, $B(-0.326, -0.946)$, $C(0.819, -0.574)$

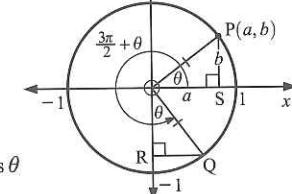
θ (degrees)	0°	90°	180°	270°	360°	450°
θ (radians)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$
sine	0	1	0	-1	0	1
cosine	1	0	-1	0	1	0
tangent	0	undefined	0	undefined	0	undefined



- 4 a $P(a, b)$ is such that $[OP]$ makes an angle of θ with the positive x -axis. $\therefore a = \cos \theta$, $b = \sin \theta$.
 $[OQ]$ makes an angle of $\pi + \theta$ with the positive x -axis.
Now Δ s POS and QOR are congruent {AAcorrS}
 $\therefore OR = a$ and $RQ = b$
 $\therefore Q$ has coordinates $(-a, -b)$, and so $\sin(\pi + \theta) = -b = -\sin \theta$



- b $P(a, b)$ is such that $[OP]$ makes an angle of θ with the positive x -axis. $\therefore a = \cos \theta$, $b = \sin \theta$.
 $[OQ]$ makes an angle of $\frac{3\pi}{2} + \theta$ with the positive x -axis.
Now Δ s POS and QOR are congruent {AAcorrS}
 $\therefore OR = a$ and $RQ = b$
 $\therefore Q$ has coordinates $(b, -a)$, and so $\sin(\frac{3\pi}{2} + \theta) = -a = -\cos \theta$



- 5 a $\sin 137^\circ$
 $= \sin(180 - 137)^\circ$
 $= \sin 43^\circ$
 ≈ 0.6820
- b $\sin 59^\circ$
 $= \sin(180 - 59)^\circ$
 $= \sin 121^\circ$
 ≈ 0.8572
- c $\cos 143^\circ$
 $= -\cos(180 - 143)^\circ$
 $= -\cos 37^\circ$
 ≈ -0.7986
- d $\cos 24^\circ$
 $= -\cos(180 - 24)^\circ$
 $= -\cos 156^\circ$
 ≈ 0.9135
- e $\sin 115^\circ$
 $= \sin(180 - 115)^\circ$
 $= \sin 65^\circ$
 ≈ 0.9063
- f $\cos 132^\circ$
 $= -\cos(180 - 132)^\circ$
 $= -\cos 48^\circ$
 ≈ -0.6691
- g $\cos^2 \theta + \sin^2 \theta = 1$
 $\therefore \cos^2 \theta + \frac{1}{4} = 1$
 $\therefore \cos^2 \theta = \frac{3}{4}$
 $\therefore \cos \theta = \pm \frac{\sqrt{3}}{2}$
- h $\cos^2 \theta + \sin^2 \theta = 1$
 $\therefore \cos^2 \theta + \frac{1}{9} = 1$
 $\therefore \cos^2 \theta = \frac{8}{9}$
 $\therefore \cos \theta = \pm \frac{\sqrt{8}}{3}$
 $= \pm \frac{2\sqrt{2}}{3}$
- i $\cos^2 \theta + \sin^2 \theta = 1$
 $\therefore \cos^2 \theta + 0 = 1$
 $\therefore \cos \theta = \pm 1$
- j $\cos^2 \theta + \sin^2 \theta = 1$
 $\therefore \cos^2 \theta + 1 = 1$
 $\therefore \cos \theta = 0$

7 a $\cos^2 \theta + \sin^2 \theta = 1$
 $\therefore \frac{16}{25} + \sin^2 \theta = 1$
 $\therefore \sin^2 \theta = \frac{9}{25}$
 $\therefore \sin \theta = \pm \frac{3}{5}$

b $\cos^2 \theta + \sin^2 \theta = 1$
 $\therefore \frac{9}{16} + \sin^2 \theta = 1$
 $\therefore \sin^2 \theta = \frac{7}{16}$
 $\therefore \sin \theta = \pm \frac{\sqrt{7}}{4}$

c $\cos^2 \theta + \sin^2 \theta = 1$
 $\therefore 1 + \sin^2 \theta = 1$
 $\therefore \sin^2 \theta = 0$
 $\therefore \sin \theta = 0$

d $\cos^2 \theta + \sin^2 \theta = 1$
 $\therefore 0 + \sin^2 \theta = 1$
 $\therefore \sin^2 \theta = 1$
 $\therefore \sin \theta = \pm 1$

8 a

Quadrant	Degree measure	Radian measure	$\cos \theta$	$\sin \theta$	$\tan \theta$
1	$0 < \theta < 90$	$0 < \theta < \frac{\pi}{2}$	+ve	+ve	+ve
2	$90 < \theta < 180$	$\frac{\pi}{2} < \theta < \pi$	-ve	+ve	-ve
3	$180 < \theta < 270$	$\pi < \theta < \frac{3\pi}{2}$	-ve	-ve	+ve
4	$270 < \theta < 360$	$\frac{3\pi}{2} < \theta < 2\pi$	+ve	-ve	-ve

- b i 1 and 4
ii 2 and 3
iii 3
iv 2

9 a $\cos^2 \theta + \sin^2 \theta = 1$
 $\therefore \frac{4}{9} + \sin^2 \theta = 1$
 $\therefore \sin^2 \theta = \frac{5}{9}$
 $\therefore \sin \theta = \pm \frac{\sqrt{5}}{3}$

But θ is in quadrant 1
where $\sin \theta > 0$
 $\therefore \sin \theta = \frac{\sqrt{5}}{3}$

b $\cos^2 \theta + \sin^2 \theta = 1$
 $\therefore \cos^2 \theta + \frac{4}{25} = 1$
 $\therefore \cos^2 \theta = \frac{21}{25}$
 $\therefore \cos \theta = \pm \frac{\sqrt{21}}{5}$

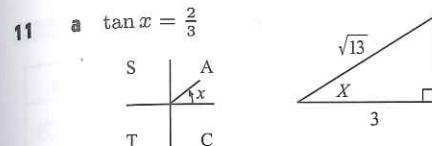
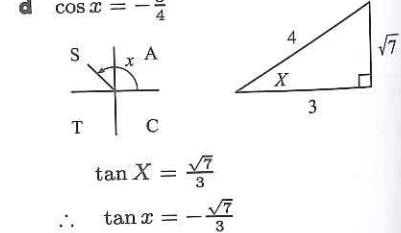
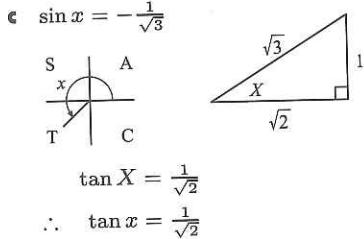
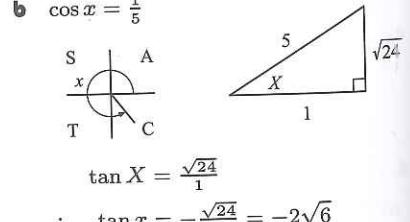
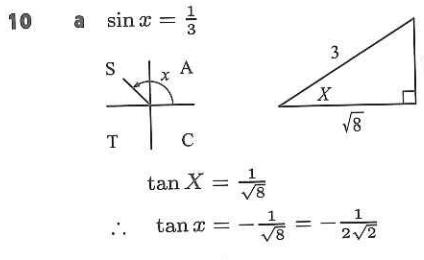
But θ is in quadrant 2
where $\cos \theta < 0$
 $\therefore \cos \theta = -\frac{\sqrt{21}}{5}$

c $\cos^2 \theta + \sin^2 \theta = 1$
 $\therefore \cos^2 \theta + \frac{9}{25} = 1$
 $\therefore \cos^2 \theta = \frac{16}{25}$
 $\therefore \cos \theta = \pm \frac{4}{5}$

But θ is in quadrant 4
where $\cos \theta > 0$
 $\therefore \cos \theta = \frac{4}{5}$

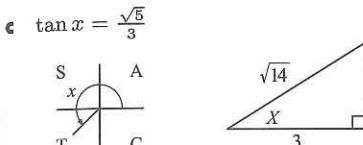
d $\cos^2 \theta + \sin^2 \theta = 1$
 $\therefore \frac{25}{169} + \sin^2 \theta = 1$
 $\therefore \sin^2 \theta = \frac{144}{169}$
 $\therefore \sin \theta = \pm \frac{12}{13}$

But θ is in quadrant 3
where $\sin \theta < 0$
 $\therefore \sin \theta = -\frac{12}{13}$



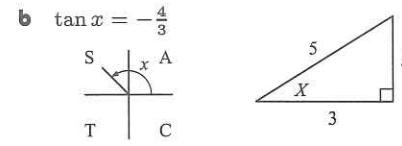
$$\sin X = \frac{2}{\sqrt{13}}, \cos X = \frac{3}{\sqrt{13}}$$

$$\therefore \sin x = \frac{2}{\sqrt{13}}, \cos x = \frac{3}{\sqrt{13}}$$



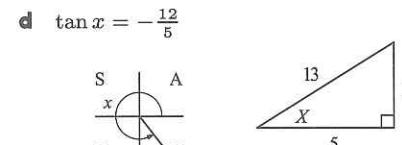
$$\sin X = \frac{\sqrt{5}}{\sqrt{14}}, \cos X = \frac{3}{\sqrt{14}}$$

$$\therefore \sin x = -\frac{\sqrt{5}}{\sqrt{14}}, \cos x = -\frac{3}{\sqrt{14}}$$



$$\sin X = \frac{4}{5}, \cos X = \frac{3}{5}$$

$$\therefore \sin x = \frac{4}{5}, \cos x = -\frac{3}{5}$$



$$\sin X = \frac{12}{13}, \cos X = \frac{5}{13}$$

$$\therefore \sin x = -\frac{12}{13}, \cos x = \frac{5}{13}$$

EXERCISE 10C.2

1 a $\sin \theta + \sin(-\theta)$
 $= \sin \theta - \sin \theta$
 $= 0$

b $\tan(-\theta) - \tan \theta$
 $= -\tan \theta - \tan \theta$
 $= -2 \tan \theta$

c $2 \cos \theta + \cos(-\theta)$
 $= 2 \cos \theta + \cos \theta$
 $= 3 \cos \theta$

d $3 \sin \theta - \sin(-\theta)$
 $= 3 \sin \theta - -\sin \theta$
 $= 3 \sin \theta + \sin \theta$
 $= 4 \sin \theta$

e $\cos^2(-\alpha)$
 $= \cos(-\alpha) \times \cos(-\alpha)$
 $= \cos \alpha \times \cos \alpha$
 $= \cos^2 \alpha$

f $\sin^2(-\alpha)$
 $= \sin(-\alpha) \times \sin(-\alpha)$
 $= -\sin \alpha \times -\sin \alpha$
 $= \sin^2 \alpha$

g $\cos(-\alpha) \cos \alpha - \sin(-\alpha) \sin \alpha$
 $= \cos \alpha \cos \alpha - -\sin \alpha \sin \alpha$
 $= \cos^2 \alpha + \sin^2 \alpha$
 $= 1$

2 a $2 \sin \theta - \cos(90^\circ - \theta)$
 $= 2 \sin \theta - \sin \theta$
 $= \sin \theta$

b $\sin(-\theta) - \cos(90^\circ - \theta)$
 $= -\sin \theta - \sin \theta$
 $= -2 \sin \theta$

c $\sin(90^\circ - \theta) - \cos \theta$
 $= \cos \theta - \cos \theta$
 $= 0$

d $3 \cos(-\theta) - 4 \sin(\frac{\pi}{2} - \theta)$
 $= 3 \cos \theta - 4 \cos \theta$
 $= -\cos \theta$

e $3 \cos \theta + \sin(\frac{\pi}{2} - \theta)$
 $= 3 \cos \theta + \cos \theta$
 $= 4 \cos \theta$

f $\cos(\frac{\pi}{2} - \theta) + 4 \sin \theta$
 $= \sin \theta + 4 \sin \theta$
 $= 5 \sin \theta$

3 $\sin(\theta - \phi) = \sin(-(\phi - \theta))$
and $\cos(\theta - \phi) = \cos(-(\phi - \theta))$
 $= -\sin(\phi - \theta)$
 $= \cos(\phi - \theta)$

4 a $\frac{\sin \theta}{\cos \theta} = \tan \theta$

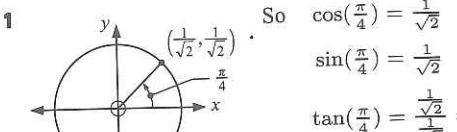
b $\frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin \theta}{\cos \theta}$
 $= -\tan \theta$

c $\frac{\sin(\frac{\pi}{2} - \theta)}{\cos(\frac{\pi}{2} - \theta)} = \frac{\cos \theta}{\cos \theta}$
 $= 1$

d $\frac{-\sin(-\theta)}{\cos \theta} = \frac{\sin \theta}{\cos \theta}$
 $= \tan \theta$

e $\frac{\cos(\frac{\pi}{2} - \theta)}{\sin(\frac{\pi}{2} - \theta)} = \frac{\sin \theta}{\cos \theta}$
 $= \tan \theta$

f $\frac{\cos(\frac{\pi}{2} - \theta)}{\cos \theta} = \frac{\sin \theta}{\cos \theta}$
 $= \tan \theta$

EXERCISE 10C.3

	a	b	c	d	e
$\sin \theta$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$
$\cos \theta$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	-1	$-\frac{1}{\sqrt{2}}$
$\tan \theta$	1	1	-1	0	1

You should draw separate unit circle diagrams for each case.

- 2 You should draw separate unit circle diagrams for each case.

3 a $\sin^2 60^\circ$
 $= \sin 60^\circ \times \sin 60^\circ$
 $= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$
 $= \frac{3}{4}$

c $4 \sin 60^\circ \cos 30^\circ$
 $= 4 \left(\frac{\sqrt{3}}{2}\right) \left(\frac{\sqrt{3}}{2}\right)$
 $= 3$

e $\sin^2(\frac{2\pi}{3}) - 1$
 $= (\frac{\sqrt{3}}{2})^2 - 1$
 $= \frac{3}{4} - 1$
 $= -\frac{1}{4}$

g $\sin(\frac{3\pi}{4}) - \cos(\frac{5\pi}{4})$
 $= \frac{1}{\sqrt{2}} - (-\frac{1}{\sqrt{2}})$
 $= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$
 $= \frac{2}{\sqrt{2}} \text{ or } \sqrt{2}$

i $\cos^2(\frac{5\pi}{6}) - \sin^2(\frac{5\pi}{6})$
 $= (-\frac{\sqrt{3}}{2})^2 - (\frac{1}{2})^2$
 $= \frac{3}{4} - \frac{1}{4}$
 $= \frac{1}{2}$

k $2 \tan(-\frac{5\pi}{4}) - \sin(\frac{3\pi}{2})$
 $= 2(-1) - (-1)$
 $= -1$

(0, -1)

b $\sin 30^\circ \cos 60^\circ$
 $= \frac{1}{2} \times \frac{1}{2}$
 $= \frac{1}{4}$

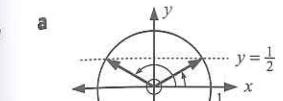
d $1 - \cos^2(\frac{\pi}{6})$
 $= 1 - (\frac{\sqrt{3}}{2})^2$
 $= 1 - \frac{3}{4}$
 $= \frac{1}{4}$

f $\cos^2(\frac{\pi}{4}) - \sin(\frac{7\pi}{6})$
 $= (\frac{1}{\sqrt{2}})^2 - (-\frac{1}{2})$
 $= \frac{1}{2} + \frac{1}{2}$
 $= 1$

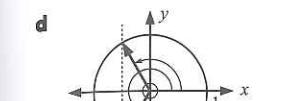
h $1 - 2 \sin^2(\frac{7\pi}{6})$
 $= 1 - 2(-\frac{1}{2})^2$
 $= 1 - 2 \times \frac{1}{4}$
 $= \frac{1}{2}$

j $\tan^2(\frac{\pi}{3}) - 2 \sin^2(\frac{\pi}{4})$
 $= (\sqrt{3})^2 - 2(\frac{1}{\sqrt{2}})^2$
 $= 3 - 2(\frac{1}{2})$
 $= 2$

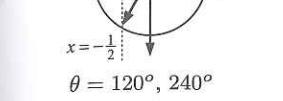
l $\frac{2 \tan 150^\circ}{1 - \tan^2 150^\circ}$
 $= \frac{2(-\frac{1}{\sqrt{3}})}{1 - (-\frac{1}{\sqrt{3}})^2}$
 $= \frac{-\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}}$
 $= \frac{-\frac{2}{\sqrt{3}}}{\frac{2}{3}} = -\frac{3}{\sqrt{3}} = -\sqrt{3}$



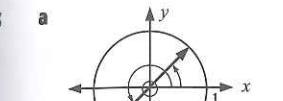
$\theta = 30^\circ, 150^\circ$



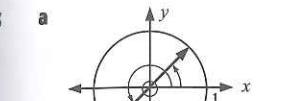
$\theta = 60^\circ, 120^\circ$



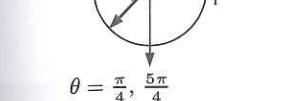
$\theta = 45^\circ, 315^\circ$



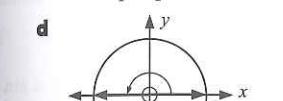
$\theta = 135^\circ, 225^\circ$



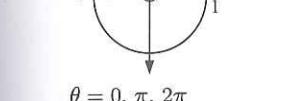
$\theta = 240^\circ, 300^\circ$



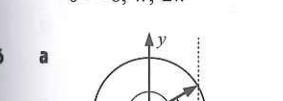
$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$



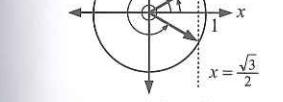
$\theta = 0, \pi, 2\pi$



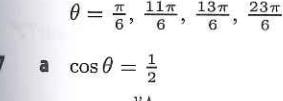
$\theta = \frac{\pi}{6}, \frac{7\pi}{6}$



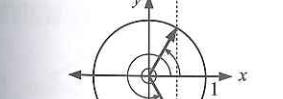
$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$



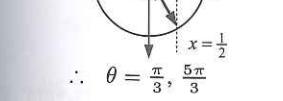
$\theta = \frac{3\pi}{2}, \frac{7\pi}{2}$



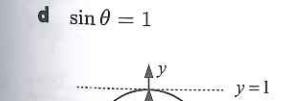
$\theta = \frac{3\pi}{2}, \frac{7\pi}{2}$



$\theta = \frac{3\pi}{2}, \frac{7\pi}{2}$



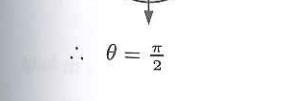
$\theta = \frac{3\pi}{2}, \frac{7\pi}{2}$



$\theta = \frac{3\pi}{2}, \frac{7\pi}{2}$



$\theta = \frac{3\pi}{2}, \frac{7\pi}{2}$



$\theta = \frac{3\pi}{2}, \frac{7\pi}{2}$



$\theta = \frac{3\pi}{2}, \frac{7\pi}{2}$



$\theta = \frac{3\pi}{2}, \frac{7\pi}{2}$

g $\cos^2 \theta = 1$
 $\therefore \cos \theta = \pm 1$

 $x = -1$ $x = 1$
 $\therefore \theta = 0, \pi, 2\pi$

h $\cos^2 \theta = \frac{1}{2}$
 $\therefore \cos \theta = \pm \frac{1}{\sqrt{2}}$

 $x = -\frac{1}{\sqrt{2}}$ $x = \frac{1}{\sqrt{2}}$
 $\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

i $\tan \theta = -\frac{1}{\sqrt{3}}$

 $\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$

j $\tan^2 \theta = 3$
 $\therefore \tan \theta = \pm \sqrt{3}$

 $\therefore \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

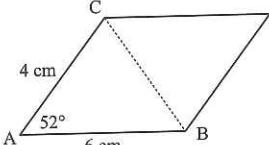
EXERCISE 10D

1 a area
 $= \frac{1}{2} \times 9 \times 10 \times \sin 40^\circ$
 $\approx 28.9 \text{ cm}^2$

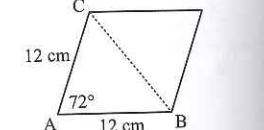
b area
 $= \frac{1}{2} \times 25 \times 31 \times \sin 82^\circ$
 $\approx 384 \text{ km}^2$

c area
 $= \frac{1}{2} \times 10.2 \times 6.4 \times \sin \frac{2\pi}{3}$
 $\approx 28.3 \text{ cm}^2$

2 area $= 150 \text{ cm}^2$
 $\therefore \frac{1}{2} \times 17 \times x \times \sin 68^\circ = 150$
 $\therefore x = \frac{2 \times 150}{17 \times \sin 68^\circ}$
 $\therefore x \approx 19.0$



area
 $= 2 \times \text{area } \triangle ABC$
 $= 2 \times \frac{1}{2} \times 4 \times 6 \times \sin 52^\circ$
 $\approx 18.9 \text{ cm}^2$



area
 $= 2 \times \text{area } \triangle ABC$
 $= 2 \times \frac{1}{2} \times 12^2 \times \sin 72^\circ$
 $\approx 137 \text{ cm}^2$

5 Regular hexagon inscribed in a circle with radius 12 cm. Central angle between two adjacent vertices is 60°.

area $= 6 \times \text{area of } \triangle$
 $= 6 \times \frac{1}{2} \times 12^2 \times \sin 60^\circ$
 $\approx 374 \text{ cm}^2$

6 Regular pentagon inscribed in a circle with side length x cm. Central angle between two adjacent vertices is 72°.

area $= 2 \times \frac{1}{2}x^2 \sin 63^\circ$
 $\therefore x^2 \sin 63^\circ = 50$
 $\therefore x^2 = \frac{50}{\sin 63^\circ}$
 $\therefore x = \sqrt{\frac{50}{\sin 63^\circ}}$
 $\therefore x \approx 7.49$
 So, sides are 7.49 cm long.

7 Equilateral triangle inscribed in a circle with side length x m. Central angle between two adjacent vertices is 72°.

area of $\triangle = \frac{338}{5}$
 $\therefore \frac{1}{2}x^2 \sin 72^\circ = \frac{338}{5}$
 $\therefore x^2 = \frac{2 \times 338}{5 \times \sin 72^\circ}$
 $\therefore x = \sqrt{\frac{2 \times 338}{5 \times \sin 72^\circ}}$
 $\therefore x \approx 11.9$
 So, OA ≈ 11.9 m long.

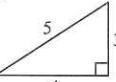
8 a If the included angle is θ
 then $\frac{1}{2} \times 5 \times 8 \times \sin \theta = 15$
 $\therefore 20 \sin \theta = 15$
 $\therefore \sin \theta = \frac{3}{4}$
 Now $\arcsin(\frac{3}{4}) \approx 48.6^\circ$
 $\therefore \theta \approx 48.6^\circ \text{ or } (180 - 48.6)^\circ$
 $\therefore \theta \approx 48.6^\circ \text{ or } 131.4^\circ$

9 Sector of a circle with radius r and central angle 360°/12° = 30°.

$\text{total area of 8 coins}$ $= 8 \times 12 \times \frac{1}{2}r^2 \sin 30^\circ$ $= 48r^2(\frac{1}{2})$ $= 24r^2$	$\text{area of \$10 note}$ $= 8r \times 4r$ $= 32r^2$	fraction covered $= \frac{24r^2}{32r^2}$ $= \frac{3}{4} \therefore \frac{1}{4} \text{ is uncovered}$
--	---	---

10 a shaded area
 $= \text{area of sector} - \text{area of triangle}$
 $= \frac{1}{2} \times 1.5 \times 12^2 - \frac{1}{2} \times 12^2 \times \sin(1.5)$
 $\approx 36.2 \text{ cm}^2$

11 a **i** area $= \frac{1}{2} \text{base} \times \text{altitude}$
 $= \frac{1}{2} \times 4 \times 3$
 $= 6 \text{ cm}^2$



ii $s = \frac{3+4+5}{2} = 6$
 $\therefore \text{area} = \sqrt{6(6-3)(6-4)(6-5)}$
 $= \sqrt{6 \times 3 \times 2 \times 1}$
 $= 6 \text{ cm}^2$

b **i** $s = \frac{6+8+12}{2} = 13$
 $\therefore A = \sqrt{13(13-6)(13-8)(13-12)}$
 $= \sqrt{13 \times 7 \times 5 \times 1}$
 $\approx 21.3 \text{ cm}^2$

ii $s = \frac{7.2+8.9+9.7}{2} = 12.9$
 $\therefore A = \sqrt{12.9(12.9-7.2)(12.9-8.9)(12.9-9.7)}$
 $= \sqrt{12.9 \times 5.7 \times 4 \times 3.2}$
 $\approx 30.7 \text{ cm}^2$

REVIEW SET 10A

1 area $= \frac{1}{2} \times 7.3 \times 9.4 \times \sin 38^\circ$
 $\approx 21.1 \text{ km}^2$

2 a area $= (\frac{80}{360}) \times \pi \times 13^2 \approx 118 \text{ cm}^2$

b area $= \frac{1}{2} \times 11 \times 9 \times \sin 65^\circ \approx 44.9 \text{ cm}^2$

3 M($\cos 73^\circ, \sin 73^\circ$) $\approx (0.292, 0.956)$, N($\cos 190^\circ, \sin 190^\circ$) $\approx (-0.985, -0.174)$, P($\cos(-53^\circ), \sin(-53^\circ)$) $\approx (0.602, -0.799)$

4 The x-coordinate of A $= -0.222$

$\therefore \cos \theta = -0.222$

$\therefore \theta = \cos^{-1}(-0.222)$

$\therefore \theta \approx 102.8^\circ, 257.2^\circ$

5 a $\sin \frac{2\pi}{3} = \sin(\pi - \frac{2\pi}{3}) = \sin \frac{\pi}{3}$

$\therefore \theta = \frac{\pi}{3}$

b $\sin 165^\circ = \sin(180 - 165)^\circ = \sin 15^\circ$

$\therefore \theta = 15^\circ$

c $\cos 276^\circ = \cos(360 - 276)^\circ = \cos 84^\circ$

$\therefore \theta = 84^\circ$

6 a $\sin 47^\circ$
 $= \sin(180 - 47)^\circ$
 $= \sin 133^\circ$
 $\therefore \theta = 133^\circ$

b $\sin \frac{\pi}{15}$
 $= \sin(\pi - \frac{\pi}{15})$
 $= \sin \frac{14\pi}{15}$
 $\therefore \theta = \frac{14\pi}{15}$

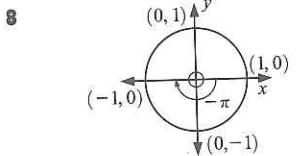
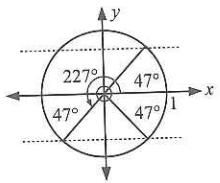
c $\cos 186^\circ$
 $= \cos(360 - 186)^\circ$
 $= \cos 174^\circ$
 $\therefore \theta = 174^\circ$

7 a $\sin 159^\circ$
 $= \sin(180 - 159)^\circ$
 $= \sin 21^\circ$
 ≈ 0.358

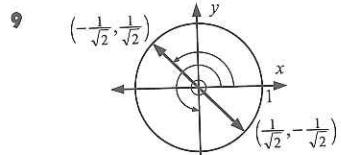
b $\cos 92^\circ$
 $= -\cos(180 - 92)^\circ$
 $= -\cos 88^\circ$
 ≈ -0.035

c $\cos 75^\circ$
 $= -\cos(180 - 75)^\circ$
 $= -\cos 105^\circ$
 ≈ 0.259

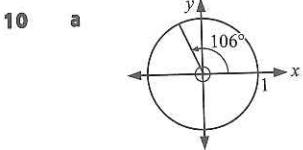
d $\sin 227^\circ = \sin(-47)^\circ$
 $= -\sin 47^\circ$
 ≈ -0.731



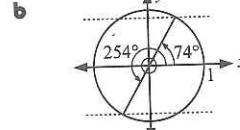
a $\cos 360^\circ = 1, \sin 360^\circ = 0$
b $\cos(-\pi) = -1, \sin(-\pi) = 0$



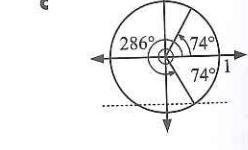
when $\cos \theta = -\sin \theta$
 $\frac{\sin \theta}{\cos \theta} = -1$ and this only occurs at the two points shown.
 $\therefore \tan \theta = -1$ So, $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$



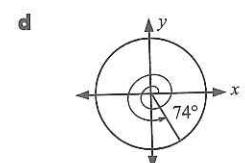
a $\sin 106^\circ$
 $= \sin(180 - 106)^\circ$
 $= \sin 74^\circ$
 ≈ 0.961



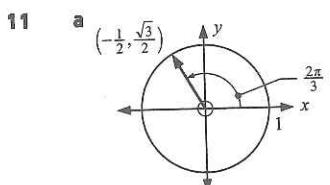
b $254^\circ = 74^\circ + 180^\circ$
 $\therefore \sin 254^\circ = -\sin 74^\circ$
 ≈ -0.961



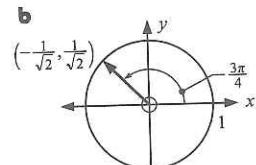
c $\sin 286^\circ = -\sin 74^\circ$
 ≈ -0.961



d $646^\circ = 360^\circ + 286^\circ$
 $\sin 646^\circ = \sin 286^\circ$
 $\approx -0.961 \quad \text{from c}$

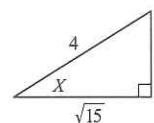
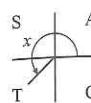


a $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$
 $\tan^2(\frac{2\pi}{3})$
 $= (-\sqrt{3})^2$
 $= 3$



b $\cos(\frac{3\pi}{4}) - \sin(\frac{3\pi}{4})$
 $= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$
 $= -\frac{2}{\sqrt{2}}$
 $= -\sqrt{2}$

12 $\sin x = -\frac{1}{4}$



$\tan X = \frac{1}{\sqrt{15}}$
 $\therefore \tan x = \frac{1}{\sqrt{15}}$

REVIEW SET 10B

1 a 120° $= (120 \times \frac{\pi}{180})^\circ$ $= \frac{2\pi}{3}^\circ$	b 225° $= 5 \times 45^\circ$ $= 5 \times \frac{\pi}{4}^\circ$ $= \frac{5\pi}{4}^\circ$	c 150° $= 5 \times 30^\circ$ $= 5 \times \frac{\pi}{6}^\circ$ $= \frac{5\pi}{6}^\circ$	d 540° $= 3 \times 180^\circ$ $= 3\pi^\circ$
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2 a 71° $= (71 \times \frac{\pi}{180})^\circ$ $\approx 1.239^\circ$	b 124.6° $= (124.6 \times \frac{\pi}{180})^\circ$ $\approx 2.175^\circ$	c -142° $= (-142 \times \frac{\pi}{180})^\circ$ $\approx -2.478^\circ$	d -25.3° $= (-25.3 \times \frac{\pi}{180})^\circ$ $\approx -0.4416^\circ$
--	--	---	--

3 a $\frac{2\pi}{5}$ $= \frac{2 \times 180^\circ}{5}$ $= 72^\circ$	b $\frac{5\pi}{4}$ $= \frac{5 \times 180^\circ}{4}$ $= 225^\circ$	c $\frac{7\pi}{9}$ $= \frac{7 \times 180^\circ}{9}$ $= 140^\circ$	d $\frac{11\pi}{6}$ $= \frac{11 \times 180^\circ}{6}$ $= 330^\circ$
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4 a 3° $= (3 \times \frac{180}{\pi})^\circ$ $\approx 171.89^\circ$	b 1.46° $= (1.46 \times \frac{180}{\pi})^\circ$ $\approx 83.65^\circ$	c 0.435° $= (0.435 \times \frac{180}{\pi})^\circ$ $\approx 24.92^\circ$	d -5.271° $= (-5.271 \times \frac{180}{\pi})^\circ$ $\approx -302.01^\circ$
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5
perimeter $= 2 \times 11 + (\frac{63}{360}) \times 2\pi \times 11$
 $\approx 34.1 \text{ cm}$ area $= (\frac{63}{360}) \times \pi \times 11^2$
 $\approx 66.5 \text{ cm}^2$

6 perimeter $= 2r + (\frac{2\pi}{3})r$ $\therefore 36 = r(2 + \frac{2\pi}{3})$ $\therefore r = \frac{36}{2 + \frac{2\pi}{3}} \text{ cm}$ $\therefore r \approx 8.79 \text{ cm}$	area $= \frac{1}{2}(\frac{2\pi}{3}) \times (8.7925)^2$ $\approx 81.0 \text{ cm}^2$
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7 area $= 42 \text{ cm}^2$ $\therefore \frac{1}{2} \times 7 \times 13 \times \sin x = 42$ $\therefore \sin x = \frac{42 \times 2}{7 \times 13}$	Now $\arcsin(\frac{42 \times 2}{7 \times 13}) \approx 67.4^\circ$ $\therefore x \approx 67.4 \text{ or } 180 - 67.4$ $\therefore x \approx 67.4 \text{ or } 112.6$
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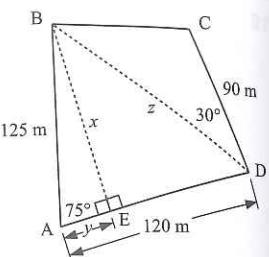
the included angle is 67.4° or 112.6° {assuming the figure is not drawn accurately}

8 a We construct a perpendicular from B to (AD).

$$\sin 75^\circ = \frac{x}{125} \therefore x \approx 120.7 \text{ m}$$

$$\cos 75^\circ = \frac{y}{125} \therefore y \approx 32.4 \text{ m}$$

$$\begin{aligned} \text{Using Pythagoras' theorem, } z^2 &= x^2 + (120 - y)^2 \\ &\approx 120.7^2 + (120 - 32.4)^2 \\ &\therefore z \approx 149.2 \text{ m} \end{aligned}$$

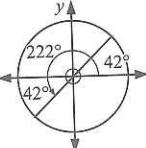


The area of the block = area of $\triangle ABD$ + area of $\triangle BCD$

$$\begin{aligned} &\approx \frac{1}{2} \times 120 \times 125 \times \sin 75^\circ + \frac{1}{2} \times 149.2 \times 90 \times \sin 30^\circ \\ &\approx 10\,600 \text{ m}^2 \end{aligned}$$

b $\approx 1.06 \text{ ha}$ { $10\,000 \text{ m}^2 = 1 \text{ ha}$ }

9 a $\cos 138^\circ$
= $-\cos(180 - 138)^\circ$
= $-\cos 42^\circ$
 ≈ -0.743



$$\begin{aligned} \cos 222^\circ &= -\cos 42^\circ \\ &\approx -0.743 \end{aligned}$$



$$\begin{aligned} \cos 318^\circ &= \cos 42^\circ \\ &\approx 0.743 \end{aligned}$$

c

$$\cos(-222^\circ)$$

$$= -\cos 42^\circ$$

$$\approx -0.743$$

10 $\cos^2 \theta + \sin^2 \theta = 1$
 $\therefore \sin^2 \theta = \frac{7}{16}$
 $\therefore \sin \theta = \pm \frac{\sqrt{7}}{4}$

$$\therefore \sin^2 \theta = \frac{7}{16}$$

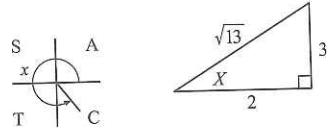
$$\therefore \sin \theta = \pm \frac{\sqrt{7}}{4}$$

11 a
 $2 \sin(\frac{\pi}{3}) \cos(\frac{\pi}{3})$
 $= 2(\frac{\sqrt{3}}{2})(\frac{1}{2})$
 $= \frac{\sqrt{3}}{2}$

b
 $\tan^2(\frac{\pi}{4}) - 1$
 $= 1^2 - 1$
 $= 0$

c
 $\cos^2(\frac{\pi}{6}) - \sin^2(\frac{\pi}{6})$
 $= (\frac{\sqrt{3}}{2})^2 - (\frac{1}{2})^2$
 $= \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

12 $\tan x = -\frac{3}{2}$



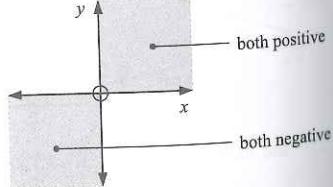
a $\sin X = \frac{3}{\sqrt{13}}$
 $\therefore \sin x = -\frac{3}{\sqrt{13}}$

b $\cos X = \frac{2}{\sqrt{13}}$
 $\therefore \cos x = \frac{2}{\sqrt{13}}$

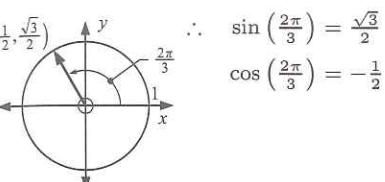
REVIEW SET 10C

1 a The point is $(\cos 320^\circ, \sin 320^\circ)$
 $\approx (0.766, -0.643)$.

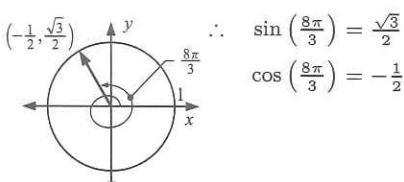
b The point is $(\cos 163^\circ, \sin 163^\circ)$
 $\approx (-0.956, 0.292)$.



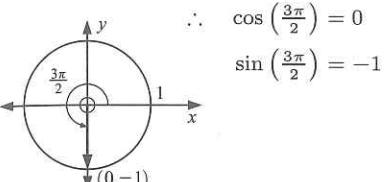
3 a $(-\frac{1}{2}, \frac{\sqrt{3}}{2}) \therefore \sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}$
 $\cos(\frac{2\pi}{3}) = -\frac{1}{2}$



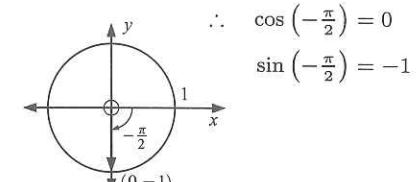
b $(-\frac{1}{2}, \frac{\sqrt{3}}{2}) \therefore \sin(\frac{8\pi}{3}) = \frac{\sqrt{3}}{2}$
 $\cos(\frac{8\pi}{3}) = -\frac{1}{2}$



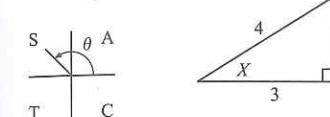
4 a $(0, -1) \therefore \cos(\frac{3\pi}{2}) = 0$
 $\sin(\frac{3\pi}{2}) = -1$



b $(0, -1) \therefore \cos(-\frac{\pi}{2}) = 0$
 $\sin(-\frac{\pi}{2}) = -1$

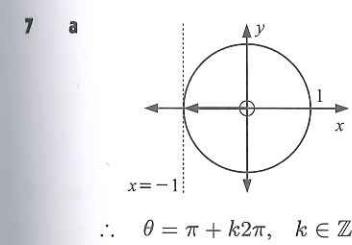
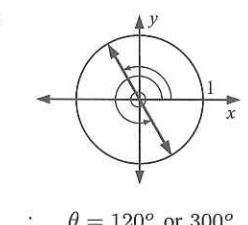
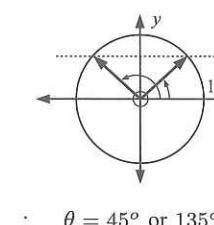
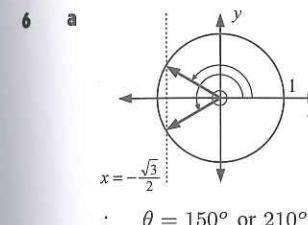


5 $\cos \theta = -\frac{3}{4}$

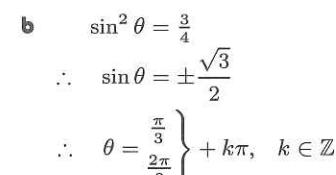


a $\sin X = \frac{\sqrt{7}}{4}$
 $\therefore \sin \theta = \frac{\sqrt{7}}{4}$

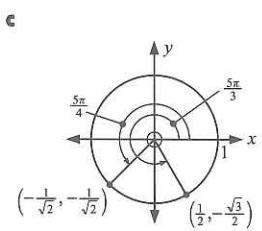
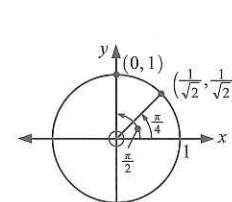
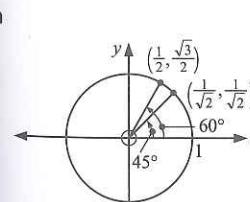
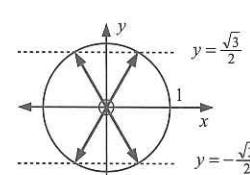
b $\tan X = \frac{\sqrt{7}}{3}$
 $\therefore \tan \theta = -\frac{\sqrt{7}}{3}$



$$\therefore \theta = \pi + k2\pi, \quad k \in \mathbb{Z}$$



$$\begin{aligned} \therefore \theta &= \frac{\pi}{3} \quad \left. \begin{array}{l} \text{or} \\ \frac{2\pi}{3} \end{array} \right\} + k\pi, \quad k \in \mathbb{Z} \end{aligned}$$



$$\begin{aligned} \tan^2 60^\circ - \sin^2 45^\circ &= (\sqrt{3})^2 - \left(\frac{1}{\sqrt{2}}\right)^2 \\ &= 3 - \frac{1}{2} \\ &= 2\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \cos^2(\frac{\pi}{4}) + \sin(\frac{\pi}{2}) &= \left(\frac{1}{\sqrt{2}}\right)^2 + 1 \\ &= \frac{1}{2} + 1 \\ &= 1\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \cos(\frac{5\pi}{3}) - \tan(\frac{5\pi}{4}) &= \frac{1}{2} - 1 \\ &= -\frac{1}{2} \end{aligned}$$

9 a $\cos\left(\frac{\pi}{2} - \theta\right) - \sin\theta = \sin\theta - \sin\theta = 0$

b $\cos\theta \tan\theta = \cos\theta \left(\frac{\sin\theta}{\cos\theta}\right) = \sin\theta$

10 area = 80 cm^2

$$\therefore \frac{1}{2} \times 11.3 \times 19.2 \sin x^\circ = 80$$

$$\therefore \sin x^\circ = \frac{160}{11.3 \times 19.2}$$

Consider the case $x \approx 47.5$.

We construct a perpendicular from A to (BC).

$$\sin 47.5^\circ = \frac{AD}{19.2} \quad \text{and} \quad \cos 47.5^\circ = \frac{BD}{19.2}$$

Using Pythagoras' theorem,

$$\begin{aligned} AC^2 &= AD^2 + CD^2 \\ &= AD^2 + (BC - BD)^2 \end{aligned}$$

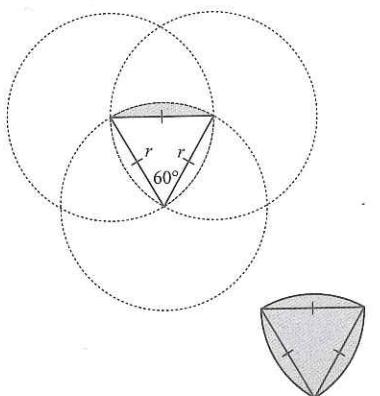
$$\therefore AC \approx \sqrt{(19.2 \sin 47.5^\circ)^2 + (11.3 - 19.2 \cos 47.5^\circ)^2} \approx 14.3 \text{ cm}$$

We can similarly construct a perpendicular from A to the extension (BC) in the case $x \approx 132.5$. In this case we obtain $AC \approx 28.1 \text{ cm}$.

11 shaded area

$$\begin{aligned} &= \text{area of sector} - \text{area of } \Delta \\ &= \frac{1}{2} \times \frac{13\pi}{18} \times 7^2 - \frac{1}{2} \times 7 \times 7 \times \sin \frac{13\pi}{18} \\ &\approx 36.8 \text{ cm}^2 \end{aligned}$$

12

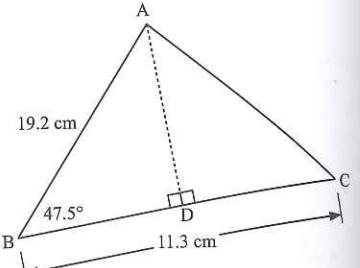


$$\begin{aligned} &\text{shaded area of sector} \\ &= \text{area of sector} - \text{area of } \Delta \\ &= \frac{60}{360} \pi r^2 - \frac{1}{2} \times r \times r \times \sin 60^\circ \\ &= \frac{\pi}{6} r^2 - \frac{1}{2} r^2 \left(\frac{\sqrt{3}}{2}\right) \\ &\therefore \text{shaded area of figure} \\ &= 3 \left[\frac{\pi}{6} r^2 - \frac{\sqrt{3}}{4} r^2 \right] + \frac{1}{2} r^2 \left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{\pi}{2} r^2 - \frac{3\sqrt{3}}{4} r^2 + \frac{\sqrt{3}}{4} r^2 \\ &= \frac{\pi}{2} r^2 - \frac{1}{2} \sqrt{3} r^2 \\ &= \frac{r^2}{2} (\pi - \sqrt{3}) \end{aligned}$$

Now $\arcsin\left(\frac{160}{11.3 \times 19.2}\right) \approx 47.5^\circ$

$$\therefore x \approx 47.5 \text{ or } 180 - 47.5$$

$$\therefore x \approx 47.5 \text{ or } 132.5$$



Chapter 11

NON-RIGHT ANGLED TRIANGLE TRIGONOMETRY

EXERCISE 11A

1 a $BC^2 = 21^2 + 15^2 - 2 \times 21 \times 15 \times \cos 105^\circ$

$$\therefore BC = \sqrt{21^2 + 15^2 - 2 \times 21 \times 15 \times \cos 105^\circ} \approx 28.8 \text{ cm}$$

b $PQ^2 = 6.3^2 + 4.8^2 - 2 \times 6.3 \times 4.8 \times \cos 32^\circ$

$$\therefore PQ = \sqrt{6.3^2 + 4.8^2 - 2 \times 6.3 \times 4.8 \times \cos 32^\circ} \approx 3.38 \text{ km}$$

c $KM^2 = 6.2^2 + 14.8^2 - 2 \times 6.2 \times 14.8 \times \cos 72^\circ$

$$\therefore KM = \sqrt{6.2^2 + 14.8^2 - 2 \times 6.2 \times 14.8 \times \cos 72^\circ} \approx 14.2 \text{ m}$$

2 $\cos A = \frac{12^2 + 13^2 - 11^2}{2 \times 12 \times 13}$

$$\therefore A = \cos^{-1}\left(\frac{192}{312}\right)$$

$$\therefore A \approx 52.0^\circ$$

$$\cos B = \frac{13^2 + 11^2 - 12^2}{2 \times 13 \times 11}$$

$$\therefore B = \cos^{-1}\left(\frac{146}{286}\right)$$

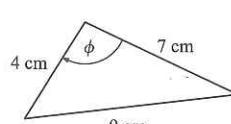
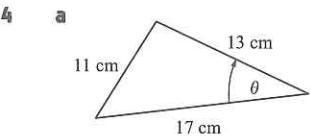
$$\therefore B \approx 59.3^\circ$$

$$\begin{aligned} C &= 180^\circ - A - B \\ &\approx 68.7^\circ \end{aligned}$$

3 $\cos Q = \frac{5^2 + 7^2 - 10^2}{2 \times 5 \times 7}$

$$\therefore Q = \cos^{-1}\left(\frac{-26}{70}\right)$$

$$\therefore Q \approx 112^\circ$$



The smallest angle is opposite the shortest side.

$$\cos \theta = \frac{13^2 + 17^2 - 11^2}{2 \times 13 \times 17}$$

$$\therefore \theta = \cos^{-1}\left(\frac{337}{442}\right)$$

$$\therefore \theta \approx 40.3$$

So, the smallest angle measures 40.3° .

The largest angle is opposite the longest side.

$$\cos \phi = \frac{4^2 + 7^2 - 9^2}{2 \times 4 \times 7}$$

$$\therefore \phi = \cos^{-1}\left(-\frac{16}{56}\right)$$

$$\therefore \phi \approx 106.60$$

So, the largest angle measures about 107° .

5 a $\cos \theta = \frac{2^2 + 5^2 - 4^2}{2 \times 2 \times 5}$

$$= \frac{13}{20}$$

$$= 0.65$$

b $x^2 = 5^2 + 3^2 - 2 \times 5 \times 3 \times \cos \theta$

$$\therefore x = \sqrt{5^2 + 3^2 - 2 \times 5 \times 3 \times 0.65}$$

$$\therefore x \approx 3.81$$

6 a $7^2 = x^2 + 6^2 - 2 \times x \times 6 \times \cos 60^\circ$

$$\therefore 49 = x^2 + 36 - 12x \times \left(\frac{1}{2}\right)$$

$$\therefore x^2 - 6x - 13 = 0$$

$$\therefore x = \frac{6 \pm \sqrt{36 - 4(1)(-13)}}{2}$$

$$= \frac{6 \pm \sqrt{88}}{2}$$

$$= 3 \pm \sqrt{22}$$

But $x > 0$, so $x = 3 + \sqrt{22}$.

b $5^2 = x^2 + 3^2 - 2 \times x \times 3 \times \cos 120^\circ$

$$\therefore 25 = x^2 + 9 - 6x \times \left(-\frac{1}{2}\right)$$

$$\therefore x^2 + 3x - 16 = 0$$

$$\therefore x = \frac{-3 \pm \sqrt{9 - 4(1)(-16)}}{2}$$

$$= \frac{-3 \pm \sqrt{73}}{2}$$

$$= \frac{-3 + \sqrt{73}}{2}$$

But $x > 0$, so $x = \frac{-3 + \sqrt{73}}{2}$.