

9 a  $\cos\left(\frac{\pi}{2} - \theta\right) - \sin\theta = \sin\theta - \sin\theta = 0$

b  $\cos\theta \tan\theta = \cos\theta \left(\frac{\sin\theta}{\cos\theta}\right) = \sin\theta$

10 area =  $80 \text{ cm}^2$   
 $\therefore \frac{1}{2} \times 11.3 \times 19.2 \sin x^\circ = 80$   
 $\therefore \sin x^\circ = \frac{160}{11.3 \times 19.2}$

Now  $\arcsin\left(\frac{160}{11.3 \times 19.2}\right) \approx 47.5^\circ$   
 $\therefore x \approx 47.5^\circ \text{ or } 180 - 47.5^\circ$   
 $\therefore x \approx 47.5^\circ \text{ or } 132.5^\circ$

Consider the case  $x \approx 47.5^\circ$ .  
We construct a perpendicular from A to (BC).

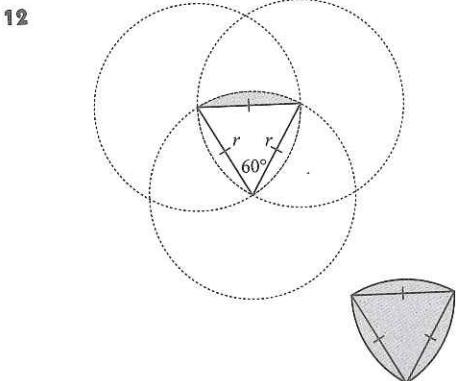
$$\sin 47.5^\circ = \frac{AD}{19.2} \quad \text{and} \quad \cos 47.5^\circ = \frac{BD}{19.2}$$

Using Pythagoras' theorem,

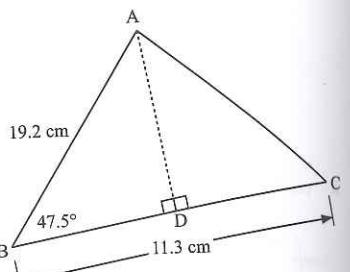
$$\begin{aligned} AC^2 &= AD^2 + CD^2 \\ &= AD^2 + (BC - BD)^2 \\ \therefore AC &\approx \sqrt{(19.2 \sin 47.5^\circ)^2 + (11.3 - 19.2 \cos 47.5^\circ)^2} \\ &\approx 14.3 \text{ cm} \end{aligned}$$

We can similarly construct a perpendicular from A to the extension (BC) in the case  $x \approx 132.5^\circ$ . In this case we obtain  $AC \approx 28.1 \text{ cm}$ .

11 shaded area  
= area of sector - area of  $\Delta$   
=  $\frac{1}{2} \times \frac{13\pi}{18} \times 7^2 - \frac{1}{2} \times 7 \times 7 \times \sin \frac{13\pi}{18}$   
 $\approx 36.8 \text{ cm}^2$



$$\begin{aligned} &\text{shaded area of sector} \\ &= \text{area of sector} - \text{area of } \Delta \\ &= \frac{60}{360} \pi r^2 - \frac{1}{2} \times r \times r \times \sin 60^\circ \\ &= \frac{\pi}{6} r^2 - \frac{1}{2} r^2 \left(\frac{\sqrt{3}}{2}\right) \\ &\therefore \text{shaded area of figure} \\ &= 3 \left[ \frac{\pi}{6} r^2 - \frac{\sqrt{3}}{4} r^2 \right] + \frac{1}{2} r^2 \left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{\pi}{2} r^2 - \frac{3\sqrt{3}}{4} r^2 + \frac{\sqrt{3}}{4} r^2 \\ &= \frac{\pi}{2} r^2 - \frac{1}{2} \sqrt{3} r^2 \\ &= \frac{r^2}{2} (\pi - \sqrt{3}) \end{aligned}$$



# Chapter 11

## NON-RIGHT ANGLED TRIANGLE TRIGONOMETRY

### EXERCISE 11A

1 a  $BC^2 = 21^2 + 15^2 - 2 \times 21 \times 15 \times \cos 105^\circ$

$$\therefore BC = \sqrt{21^2 + 15^2 - 2 \times 21 \times 15 \times \cos 105^\circ} \approx 28.8 \text{ cm}$$

b  $PQ^2 = 6.3^2 + 4.8^2 - 2 \times 6.3 \times 4.8 \times \cos 32^\circ$

$$\therefore PQ = \sqrt{6.3^2 + 4.8^2 - 2 \times 6.3 \times 4.8 \times \cos 32^\circ} \approx 3.38 \text{ km}$$

c  $KM^2 = 6.2^2 + 14.8^2 - 2 \times 6.2 \times 14.8 \times \cos 72^\circ$

$$\therefore KM = \sqrt{6.2^2 + 14.8^2 - 2 \times 6.2 \times 14.8 \times \cos 72^\circ} \approx 14.2 \text{ m}$$

2  $\cos A = \frac{12^2 + 13^2 - 11^2}{2 \times 12 \times 13}$

$$\cos B = \frac{13^2 + 11^2 - 12^2}{2 \times 13 \times 11}$$

$$\begin{aligned} C &= 180^\circ - A - B \\ &\approx 68.7^\circ \end{aligned}$$

$$\therefore A = \cos^{-1}\left(\frac{192}{312}\right)$$

$$\therefore B = \cos^{-1}\left(\frac{146}{286}\right)$$

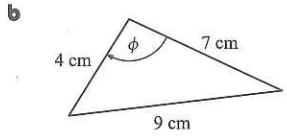
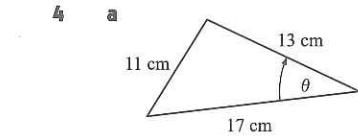
$$\therefore A \approx 52.0^\circ$$

$$\therefore B \approx 59.3^\circ$$

3  $\cos Q = \frac{5^2 + 7^2 - 10^2}{2 \times 5 \times 7}$

$$\therefore Q = \cos^{-1}\left(-\frac{26}{70}\right)$$

$$\therefore Q \approx 112^\circ$$



The smallest angle is opposite the shortest side.

$$\cos \theta = \frac{13^2 + 17^2 - 11^2}{2 \times 13 \times 17}$$

$$\therefore \theta = \cos^{-1}\left(\frac{337}{442}\right)$$

$$\therefore \theta \approx 40.3$$

So, the smallest angle measures  $40.3^\circ$ .

The largest angle is opposite the longest side.

$$\cos \phi = \frac{4^2 + 7^2 - 9^2}{2 \times 4 \times 7}$$

$$\therefore \phi = \cos^{-1}\left(-\frac{16}{56}\right)$$

$$\therefore \phi \approx 106.60$$

So, the largest angle measures about  $107^\circ$ .

5 a  $\cos \theta = \frac{2^2 + 5^2 - 4^2}{2 \times 2 \times 5}$   
 $= \frac{13}{20}$   
 $= 0.65$

b  $x^2 = 5^2 + 3^2 - 2 \times 5 \times 3 \times \cos \theta$   
 $\therefore x = \sqrt{5^2 + 3^2 - 2 \times 5 \times 3 \times 0.65}$   
 $\therefore x \approx 3.81$

6 a  $7^2 = x^2 + 6^2 - 2 \times x \times 6 \times \cos 60^\circ$   
 $\therefore 49 = x^2 + 36 - 12x \times \left(\frac{1}{2}\right)$   
 $\therefore x^2 - 6x - 13 = 0$

$$\begin{aligned} \therefore x &= \frac{6 \pm \sqrt{36 - 4(1)(-13)}}{2} \\ &= \frac{6 \pm \sqrt{88}}{2} \\ &= 3 \pm \sqrt{22} \end{aligned}$$

But  $x > 0$ , so  $x = 3 + \sqrt{22}$ .

b  $5^2 = x^2 + 3^2 - 2 \times x \times 3 \times \cos 120^\circ$   
 $\therefore 25 = x^2 + 9 - 6x \times \left(-\frac{1}{2}\right)$   
 $\therefore x^2 + 3x - 16 = 0$

$$\begin{aligned} \therefore x &= \frac{-3 \pm \sqrt{9 - 4(1)(-16)}}{2} \\ &= \frac{-3 \pm \sqrt{73}}{2} \\ &= \frac{-3 + \sqrt{73}}{2} \end{aligned}$$

But  $x > 0$ , so  $x = \frac{-3 + \sqrt{73}}{2}$ .

c  $5^2 = (2x)^2 + x^2 - 2 \times (2x) \times x \times \cos 60^\circ$   
 $\therefore 25 = 4x^2 + x^2 - 4x^2(\frac{1}{2})$   
 $\therefore 3x^2 = 25$   
 $\therefore x^2 = \frac{25}{3}$   
 $\therefore x = \pm \frac{5}{\sqrt{3}}$  But  $x > 0$ , so  $x = \frac{5}{\sqrt{3}}$ .

7 a  $11^2 = x^2 + 8^2 - 2 \times x \times 8 \times \cos 70^\circ$

$\therefore 121 = x^2 + 64 - 16x \cos 70^\circ$   
 $\therefore x^2 - (16 \cos 70^\circ)x - 57 = 0$

Using the quadratic formula or technology,

$x \approx -5.29$  or  $10.8$ .

But  $x > 0$ , so  $x \approx 10.8$ .

8 a  $(3x+1)^2 = (x+2)^2 + (x+3)^2 - 2(x+2)(x+3) \cos \theta$

$\therefore 9x^2 + 6x + 1 = x^2 + 4x + 4 + x^2 + 6x + 9 - 2(x^2 + 5x + 6)(-\frac{1}{5})$

$\therefore 9x^2 + 6x + 1 = 2x^2 + 10x + 13 + \frac{2}{5}x^2 + 2x + \frac{12}{5}$

$\therefore \frac{33}{5}x^2 - 6x - \frac{72}{5} = 0$

$\therefore 33x^2 - 30x - 72 = 0$

$\therefore 3(11x+12)(x-2) = 0$

$\therefore x = -\frac{12}{11}$  or  $2$

But  $3x+1 > 0$ , so  $x = 2$ .

b  $\cos^2 \theta + \sin^2 \theta = 1$

$\therefore \frac{1}{25} + \sin^2 \theta = 1$

$\therefore \sin^2 \theta = \frac{24}{25}$

$\therefore \sin \theta = \pm \frac{\sqrt{24}}{5}$

But  $0^\circ < \theta < 180^\circ$ , so  $\sin \theta > 0$ .

$\therefore \sin \theta = \frac{\sqrt{24}}{5}$

b  $13^2 = x^2 + 5^2 - 2 \times x \times 5 \times \cos 130^\circ$

$\therefore 169 = x^2 + 25 - 10x \cos 130^\circ$

$\therefore x^2 - (10 \cos 130^\circ)x - 144 = 0$

Using the quadratic formula or technology,

$x \approx -15.6$  or  $9.21$ .

But  $x > 0$ , so  $x \approx 9.21$ .

8 a

$(3x+1)^2 = (x+2)^2 + (x+3)^2 - 2(x+2)(x+3) \cos \theta$

$\therefore 9x^2 + 6x + 1 = x^2 + 4x + 4 + x^2 + 6x + 9 - 2(x^2 + 5x + 6)(-\frac{1}{5})$

$\therefore 9x^2 + 6x + 1 = 2x^2 + 10x + 13 + \frac{2}{5}x^2 + 2x + \frac{12}{5}$

$\therefore \frac{33}{5}x^2 - 6x - \frac{72}{5} = 0$

$\therefore 33x^2 - 30x - 72 = 0$

$\therefore 3(11x+12)(x-2) = 0$

$\therefore x = -\frac{12}{11}$  or  $2$

But  $3x+1 > 0$ , so  $x = 2$ .

b  $\text{area} = \frac{1}{2} \times (x+2) \times (x+3) \times \sin \theta$

$= \frac{1}{2} \times 4 \times 5 \times \frac{\sqrt{24}}{5}$

$= 2\sqrt{24} \text{ cm}^2$

### EXERCISE 11B.1

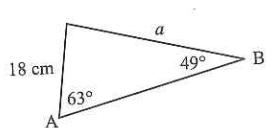
1 a By the sine rule,

$$\frac{x}{\sin 48^\circ} = \frac{23}{\sin 37^\circ}$$

$$\therefore x = \frac{23 \times \sin 48^\circ}{\sin 37^\circ}$$

$$\therefore x \approx 28.4$$

2 a



$$\frac{a}{\sin 63^\circ} = \frac{18}{\sin 49^\circ} \quad \{\text{sine rule}\}$$

$$\therefore a = \frac{18 \times \sin 63^\circ}{\sin 49^\circ}$$

$$\therefore a \approx 21.3 \text{ cm}$$

b By the sine rule,

$$\frac{x}{\sin 115^\circ} = \frac{11}{\sin 48^\circ}$$

$$\therefore x = \frac{11 \times \sin 115^\circ}{\sin 48^\circ}$$

$$\therefore x \approx 13.4$$

b  $(180 - 82 - 25)^\circ = 73^\circ$

c By the sine rule,

$$\frac{x}{\sin 51^\circ} = \frac{4.8}{\sin 80^\circ}$$

$$\therefore x = \frac{4.8 \times \sin 51^\circ}{\sin 80^\circ}$$

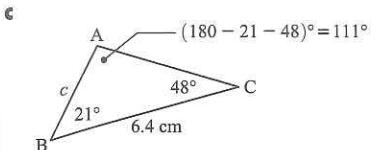
$$\therefore x \approx 3.79$$

b  $(180 - 82 - 25)^\circ = 73^\circ$

By the sine rule,  $\frac{b}{\sin 73^\circ} = \frac{34}{\sin 25^\circ}$

$$\therefore b = \frac{34 \times \sin 73^\circ}{\sin 25^\circ}$$

$$\therefore b \approx 76.9 \text{ cm}$$



By the sine rule,  $\frac{c}{\sin 48^\circ} = \frac{6.4}{\sin 111^\circ}$   
 $\therefore c = \frac{6.4 \times \sin 48^\circ}{\sin 111^\circ}$   
 $\therefore c \approx 5.09 \text{ cm}$

### EXERCISE 11B.2

1 By the sine rule,  $\frac{\sin C}{11} = \frac{\sin 40^\circ}{8}$

$$\therefore \sin C = \frac{11 \times \sin 40^\circ}{8}$$

$$\therefore C = \sin^{-1} \left( \frac{11 \times \sin 40^\circ}{8} \right) \text{ or its supplement}$$

$$\therefore C \approx 62.1^\circ \text{ or } (180 - 62.1)^\circ$$

$$\therefore C \approx 62.1^\circ \text{ or } 117.9^\circ$$

2 a  $\frac{\sin A}{a} = \frac{\sin B}{b}$

$$\therefore \sin A = \frac{14.6 \times \sin 65^\circ}{17.4}$$

$$\therefore A = \sin^{-1} \left( \frac{14.6 \times \sin 65^\circ}{17.4} \right)$$

or its supplement

$$\therefore A \approx 49.5^\circ \text{ or } 180^\circ - 49.5^\circ$$

$$\therefore A \approx 49.5^\circ \text{ or } 130.5^\circ$$

Check:  $A = 130.5^\circ$  is impossible as  $A + B = 130.5^\circ + 65^\circ$  is already over  $180^\circ$ .  $\therefore A \approx 49.5^\circ$

$$\frac{\sin C}{4.8} = \frac{\sin 71^\circ}{6.5}$$

$$\therefore \sin C = \frac{4.8 \times \sin 71^\circ}{6.5}$$

$$\therefore C = \sin^{-1} \left( \frac{4.8 \times \sin 71^\circ}{6.5} \right) \text{ or its supplement}$$

$$\therefore C \approx 44.3^\circ \text{ or } 135.7^\circ$$

But  $135.7 + 71 > 180$   $\therefore$  this case is impossible  $\therefore C \approx 44.3^\circ$

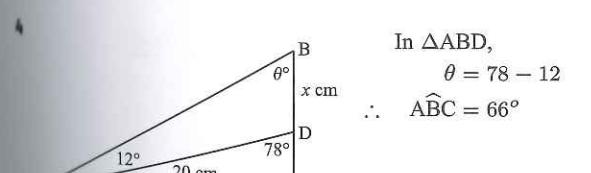
3 The third angle is  $180^\circ - 85^\circ - 68^\circ = 27^\circ$

$$\frac{\sin 85^\circ}{11.4} \text{ and } \frac{\sin 27^\circ}{9.8}$$

$$\approx 0.08738 \quad \approx 0.04632$$

$$\therefore \text{it is not possible as } \frac{\sin 85^\circ}{11.4} \neq \frac{\sin 27^\circ}{9.8}$$

$\therefore$  the sine rule is violated.



In  $\triangle ABD$ ,

$$\theta = 78 - 12$$

$$\therefore \widehat{ABC} = 66^\circ$$

$$\text{Now } \frac{x}{\sin 12^\circ} = \frac{20}{\sin 66^\circ}$$

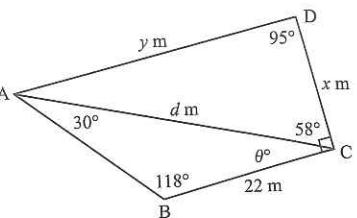
$$\therefore x = \frac{20 \times \sin 12^\circ}{\sin 66^\circ}$$

$$\therefore x \approx 4.55$$

$\therefore [BD]$  is 4.55 cm long

5 First we find the length of the diagonal,  $d$  m.

$$\begin{aligned} \frac{d}{\sin 118^\circ} &= \frac{22}{\sin 30^\circ} \\ \therefore d &= \frac{22 \times \sin 118^\circ}{\sin 30^\circ} \\ \therefore d &\approx 38.85 \end{aligned}$$



Using the sine rule

$$\begin{aligned} \frac{y}{\sin 58^\circ} &= \frac{38.85}{\sin 95^\circ} \\ \therefore y &\approx \frac{38.85 \times \sin 58^\circ}{\sin 95^\circ} \\ \therefore y &\approx 33.1 \end{aligned}$$

$$\theta = 180 - 30 - 118 = 32$$

$$\begin{aligned} \therefore \widehat{ACD} &= 58^\circ \\ \text{and } \frac{x}{\sin(180 - 95 - 58)^\circ} &\approx \frac{38.85}{\sin 95^\circ} \\ \therefore x &\approx \frac{38.85 \times \sin 27^\circ}{\sin 95^\circ} \\ \therefore x &\approx 17.7 \end{aligned}$$

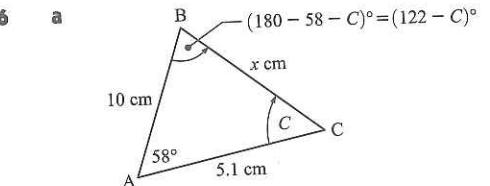
$$\frac{\sin C}{10} = \frac{\sin(122 - C)}{5.1}$$

$$\therefore 5.1 \sin C = 10 \sin(122 - C)$$

Using technology,

$$C \approx 88.7^\circ \text{ or } (180 - 88.7)^\circ$$

$$\therefore C \approx 88.7^\circ \text{ or } 91.3^\circ$$



$$\begin{aligned} \text{b} \quad \text{Let } BC = x \text{ cm} \quad \therefore x^2 &= 10^2 + 5.1^2 - 2 \times 10 \times 5.1 \cos 58^\circ \\ \therefore x &= \sqrt{10^2 + 5.1^2 - 2 \times 10 \times 5.1 \cos 58^\circ} \end{aligned}$$

$$\therefore x \approx 8.4828$$

$$\text{and } \cos C = \frac{5.1^2 + 8.4828^2 - 10^2}{2 \times 5.1 \times 8.4828} \approx -0.02309$$

$$\therefore C \approx \arccos(-0.02309) \approx 91.3^\circ$$

c "When faced with using either the sine rule or the cosine rule it is better to use the cosine rule as it avoids the ambiguous case."

$$\begin{aligned} 7 \quad 9^2 &= x^2 + 7^2 - 2 \times x \times 7 \times \cos 30^\circ \\ \therefore 81 &= x^2 + 49 - 14x\left(\frac{\sqrt{3}}{2}\right) \end{aligned}$$

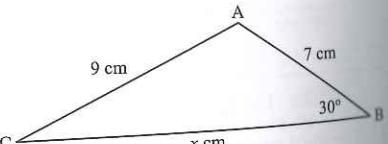
$$\therefore x^2 - \frac{14\sqrt{3}}{2}x - 32 = 0$$

Using the quadratic formula or technology,

$$x \approx -2.23 \text{ or } 14.35$$

but  $x > 0$ , so  $x \approx 14.35$

$$\therefore \text{area of triangle} \approx \frac{1}{2} \times 7 \times 14.35 \times \sin 30^\circ \approx 25.1 \text{ cm}^2$$



$$8 \quad \frac{2x - 5}{\sin 45^\circ} = \frac{x + 3}{\sin 30^\circ}$$

$$\therefore (2x - 5) \sin 30^\circ = (x + 3) \sin 45^\circ$$

$$\therefore \frac{2x - 5}{2} = \frac{x + 3}{\sqrt{2}}$$

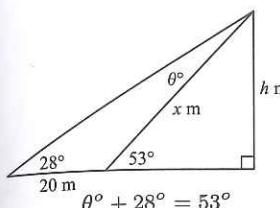
$$\therefore 2\sqrt{2}x - 5\sqrt{2} = 2x + 6$$

$$\therefore -6 - 5\sqrt{2} = x(2 - 2\sqrt{2})$$

$$\begin{aligned} \therefore x &= \left( \frac{-6 - 5\sqrt{2}}{2 - 2\sqrt{2}} \right) \left( \frac{2 + 2\sqrt{2}}{2 + 2\sqrt{2}} \right) \\ &= \frac{-12 - 12\sqrt{2} - 10\sqrt{2} - 10(2)}{4 - 4(2)} \\ &= \frac{-32 - 22\sqrt{2}}{-4} \\ &= 8 + \frac{11\sqrt{2}}{2} \end{aligned}$$

### EXERCISE 11C

1



By the sine rule,

$$\frac{x}{\sin 28^\circ} = \frac{20}{\sin 25^\circ}$$

$$\therefore x \approx \frac{20 \times \sin 28^\circ}{\sin 25^\circ}$$

$$\therefore x \approx 22.22$$

and  $\sin 53^\circ = \frac{h}{x}$

$$\therefore h = x \sin 53^\circ$$

$$\approx 22.22 \times \sin 53^\circ$$

$$\approx 17.7 \text{ m}$$

$\therefore$  the pole is 17.7 m high.

2

$$\begin{aligned} PR^2 &= 63^2 + 175^2 - 2 \times 63 \times 175 \times \cos 112^\circ \\ \therefore PR &= \sqrt{63^2 + 175^2 - 2 \times 63 \times 175 \times \cos 112^\circ} \\ \therefore PR &\approx 207 \text{ m} \end{aligned}$$

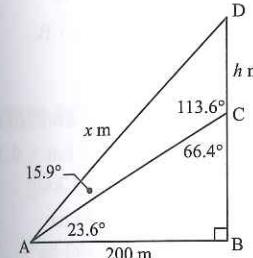
$$3 \quad \cos T = \frac{220^2 + 340^2 - 165^2}{2 \times 220 \times 340}$$

$$\therefore T = \cos^{-1} \left( \frac{136775}{149600} \right)$$

$$\therefore T \approx 23.9$$

$\therefore$  the tee shot was 23.9° off line.

4



In  $\triangle ABD$ ,

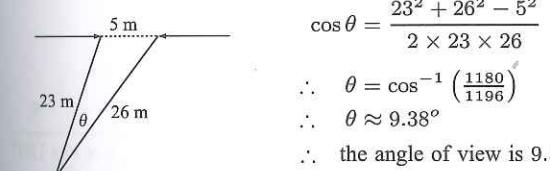
$$\begin{aligned} \cos(23.6 + 15.9)^\circ &= \frac{200}{x} \\ \therefore x &= \frac{200}{\cos 39.5^\circ} \\ \therefore x &\approx 259.2 \end{aligned}$$

In  $\triangle ACD$ ,

$$\begin{aligned} \frac{h}{\sin 15.9^\circ} &= \frac{x}{\sin 113.6^\circ} \\ \therefore h &\approx \frac{259.2 \times \sin 15.9^\circ}{\sin 113.6^\circ} \\ \therefore h &\approx 77.5 \end{aligned}$$

$\therefore$  the tower is 77.5 m high.

5



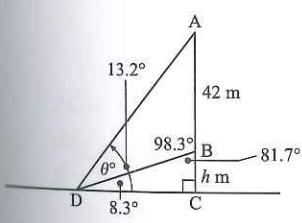
$$\cos \theta = \frac{23^2 + 26^2 - 5^2}{2 \times 23 \times 26}$$

$$\therefore \theta = \cos^{-1} \left( \frac{1180}{1196} \right)$$

$\therefore \theta \approx 9.38^\circ$

$\therefore$  the angle of view is 9.38°.

6



In  $\triangle ABD$ ,

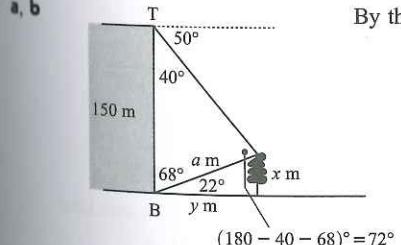
$$\begin{aligned} \frac{AD}{\sin 98.3^\circ} &= \frac{42}{\sin 4.9^\circ} \\ \therefore AD &= \frac{42 \times \sin 98.3^\circ}{\sin 4.9^\circ} \\ \therefore AD &\approx 486.56 \text{ m} \end{aligned}$$

In  $\triangle ADC$ ,

$$\begin{aligned} \frac{AD}{\sin 13.2^\circ} &= \frac{h + 42}{\sin 4.9^\circ} \\ \therefore h + 42 &\approx 486.56 \times \sin 13.2^\circ \\ \therefore h + 42 &\approx 111.1 \\ \therefore h &\approx 69.1 \end{aligned}$$

$\therefore$  the hill is 69.1 m high.

7 a, b



$$\text{By the sine rule, } \frac{a}{\sin 40^\circ} = \frac{150}{\sin 72^\circ}$$

$$\therefore a = \frac{150 \times \sin 40^\circ}{\sin 72^\circ}$$

$$\therefore a \approx 101.38$$

$$\text{Now } \sin 22^\circ \approx \frac{x}{101.38}$$

$$\therefore x \approx 101.38 \times \sin 22^\circ$$

$$\therefore x \approx 38.0$$

$$\text{and } \cos 22^\circ \approx \frac{y}{101.38}$$

$$\therefore y \approx 101.38 \times \cos 22^\circ$$

$$\therefore y \approx 94.0$$

$\therefore$  the tree is 38.0 m high and 94.0 m from the building.

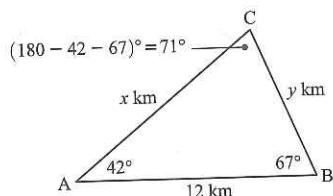
- 8 Using Pythagoras' theorem

$$RQ = \sqrt{4^2 + 7^2} = \sqrt{65} \text{ m}$$

$$PQ = \sqrt{8^2 + 7^2} = \sqrt{113} \text{ cm}$$

$$PR = \sqrt{8^2 + 4^2} = \sqrt{80} \text{ cm}$$

9

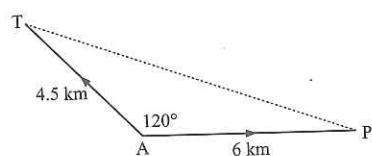


10 a

$$QS = \sqrt{8^2 + 12^2 - 2 \times 8 \times 12 \times \cos 70^\circ} \approx 11.93$$

$$\therefore \text{area} \approx \frac{1}{2} \times 8 \times 12 \times \sin 70^\circ + \frac{1}{2} \times 10 \times 11.93 \times \sin 30^\circ \approx 74.9 \text{ km}^2$$

11



Distance = speed  $\times$  time

So, after 45 min = 0.75 h

$$AT = 6 \times 0.75 = 4.5 \text{ km}$$

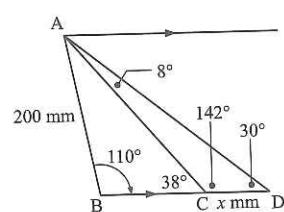
$$AP = 8 \times 0.75 = 6 \text{ km}$$

$$\text{Now } PT = \sqrt{4.5^2 + 6^2 - 2 \times 4.5 \times 6 \times \cos 120^\circ}$$

$$\therefore PT \approx 9.12$$

So, they are 9.12 km apart.

12



$$\text{In } \triangle ABC, \frac{AC}{\sin 110^\circ} = \frac{200}{\sin 38^\circ}$$

$$\therefore AC = \frac{200 \times \sin 110^\circ}{\sin 38^\circ} \approx 305.26$$

$$\text{and in } \triangle ACD, \frac{x}{\sin 8^\circ} \approx \frac{305.26}{\sin 30^\circ}$$

$$\therefore x \approx \frac{305.26 \times \sin 8^\circ}{\sin 30^\circ} \approx 84.968$$

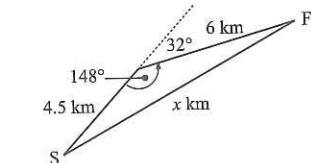
$\therefore$  the metal strip is 85.0 mm wide.

$$x = \sqrt{6^2 + (4.5)^2 - 2 \times 6 \times 4.5 \times \cos 148^\circ}$$

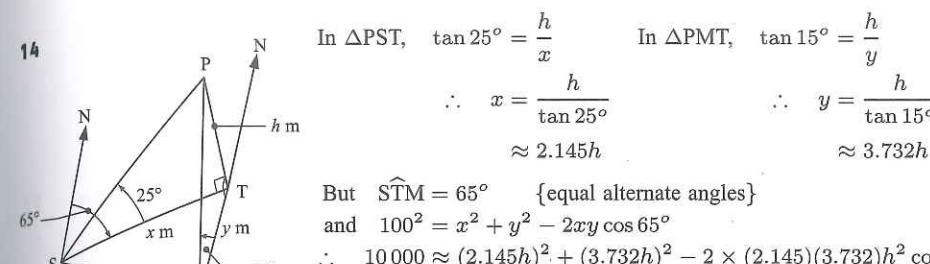
$$\therefore x \approx 10.1$$

$\therefore$  the orienteer is 10.1 km from the start.

13



14



$$\text{In } \triangle PST, \tan 25^\circ = \frac{h}{x}$$

$$\text{In } \triangle PMT, \tan 15^\circ = \frac{h}{y}$$

$$\therefore x = \frac{h}{\tan 25^\circ}$$

$$\therefore y = \frac{h}{\tan 15^\circ}$$

$$\approx 2.145h$$

$$\approx 3.732h$$

But  $\widehat{STM} = 65^\circ$  {equal alternate angles}

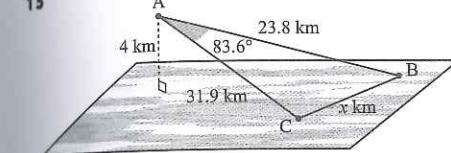
$$\text{and } 100^2 = x^2 + y^2 - 2xy \cos 65^\circ$$

$$\therefore 10000 \approx (2.145h)^2 + (3.732h)^2 - 2 \times (2.145)(3.732)h^2 \cos 65^\circ$$

$$\therefore h^2 \approx 850.17$$

$\therefore h \approx 29.2$  So, the tree is 29.2 m high.

15



By the cosine rule

$$x^2 = 23.8^2 + 31.9^2 - 2 \times 23.8 \times 31.9 \times \cos 83.6^\circ$$

$$\therefore x = \sqrt{23.8^2 + 31.9^2 - 2 \times 23.8 \times 31.9 \times \cos 83.6^\circ}$$

$$\therefore x \approx 37.6$$

B and C are 37.6 km apart.

### REVIEW SET 11A

$$1 \text{ a } \cos x^\circ = \frac{13^2 + 19^2 - 11^2}{2 \times 13 \times 19}$$

$$\therefore \cos x^\circ = \frac{409}{494}$$

$$\therefore x^\circ = \cos^{-1} \left( \frac{409}{494} \right)$$

$$\therefore x \approx 34.1$$

$$2 \text{ a } \cos x^\circ = \frac{11^2 + 19^2 - 13^2}{2 \times 11 \times 19}$$

$$\therefore \cos x^\circ = \frac{313}{418}$$

$$\therefore x^\circ = \cos^{-1} \left( \frac{313}{418} \right)$$

$$\therefore x \approx 41.5$$

$$3 \text{ a } AC = \sqrt{11^2 + 9.8^2 - 2 \times 11 \times 9.8 \times \cos 74^\circ}$$

$$\therefore AC \approx 12.554 \text{ cm}$$

$$\therefore AC \approx 12.6 \text{ cm}$$

$$\text{Now } \frac{\sin C}{11} = \frac{\sin 74^\circ}{AC}$$

$$\therefore \sin C \approx \frac{11 \times \sin 74^\circ}{12.554}$$

$$\therefore C \approx \sin^{-1} \left( \frac{11 \times \sin 74^\circ}{12.554} \right) \text{ or its supplement}$$

$$\therefore C \approx 57.4^\circ \text{ or } 122.6^\circ$$

↑

impossible as  $122.6 + 74 > 180$

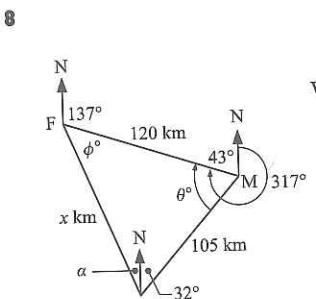
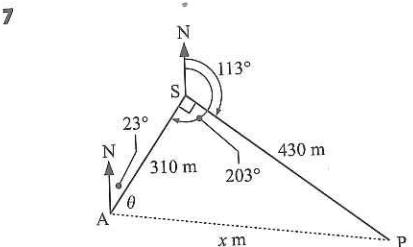
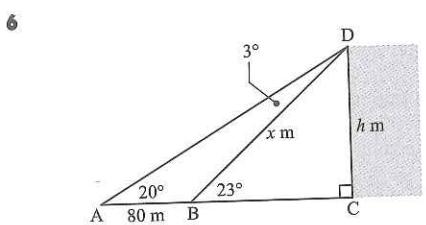
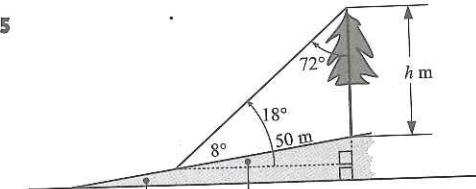
$$\therefore C \text{ measures } 57.4^\circ$$

$$\therefore A \text{ measures } 48.6^\circ$$

$$DB = \sqrt{7^2 + 11^2 - 2 \times 7 \times 11 \times \cos 110^\circ} \approx 14.922 \text{ cm}$$

$$\text{total area} \approx \frac{1}{2} \times 7 \times 11 \times \sin 110^\circ + \frac{1}{2} \times 16 \times 14.922 \times \sin 40^\circ$$

$$\approx 113 \text{ cm}^2$$



9 If the unknown is an angle, use the cosine rule to avoid the ambiguous case.

10 a By the cosine rule,  $7^2 = 8^2 + x^2 - 2 \times 8 \times x \cos 60^\circ$

$$\therefore 49 = 64 + x^2 - 16x \left(\frac{1}{2}\right)$$

$$\therefore 49 = 64 + x^2 - 8x$$

$$\therefore x^2 - 8x + 15 = 0$$

$$\therefore (x-3)(x-5) = 0$$

$$\therefore x = 3 \text{ or } 5$$

$$\begin{aligned} \frac{h}{\sin 8^\circ} &= \frac{50}{\sin 72^\circ} \\ \therefore h &= \frac{50 \times \sin 8^\circ}{\sin 72^\circ} \\ \therefore h &\approx 7.32 \end{aligned}$$

So, the tree is 7.32 m high.

$$\begin{aligned} \text{In } \triangle ABD, \frac{x}{\sin 20^\circ} &= \frac{80}{\sin 3^\circ} \\ \therefore x &= \frac{80 \times \sin 20^\circ}{\sin 3^\circ} \approx 522.8 \end{aligned}$$

$$\begin{aligned} \text{Now } \sin 23^\circ &= \frac{h}{x} \\ \therefore h &\approx 522.8 \times \sin 23^\circ \\ \therefore h &\approx 204 \end{aligned}$$

So the building is 204 m tall.

$$\begin{aligned} \widehat{ASP} &= 203^\circ - 113^\circ = 90^\circ \\ \therefore x^2 &= 310^2 + 430^2 \quad \{\text{Pythagoras}\} \\ \therefore x &= \sqrt{310^2 + 430^2} \end{aligned}$$

$$\therefore x \approx 530$$

they are 530 m apart.

$$\begin{aligned} \text{and } \tan \theta &= \frac{430}{310} \\ \therefore \theta &= \tan^{-1} \left( \frac{430}{310} \right) \approx 54.2 \end{aligned}$$

$$\text{and } 23 + \theta = 77.2$$

∴ the bearing of Peter from Alix is 077.2°.

In 45 minutes,  $140 \times \frac{3}{4} = 105 \text{ km}$  is travelled.

In 40 minutes,  $180 \times \frac{2}{3} = 120 \text{ km}$  is travelled.

We notice that  $\theta + 43 + 32 = 180$  {co-interior angles add to 180°}

$$\therefore \theta = 105$$

$$\begin{aligned} \text{and so, } x &= \sqrt{120^2 + 105^2 - 2 \times 120 \times 105 \cos 105^\circ} \\ \therefore x &\approx 178.74 \end{aligned}$$

So, the car is 179 km from the start.

$$\text{Now } \frac{\sin \phi}{105} \approx \frac{\sin 105^\circ}{178.74}$$

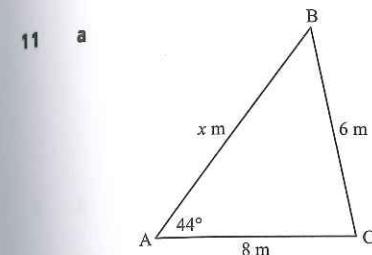
$$\therefore \sin \phi \approx \frac{105 \times \sin 105^\circ}{178.74}$$

$$\therefore \phi \approx 34.6$$

$$\therefore \alpha \approx 180 - 105 - 34.6 - 32 \approx 8.4 \approx 8$$

So, the bearing from its starting point is 352°.

- b Kady's response should be "Please supply me with additional information as there are two possibilities. Which one do you want?"



$$\text{By the cosine rule, } 6^2 = x^2 + 8^2 - 2 \times x \times 8 \times \cos 44^\circ$$

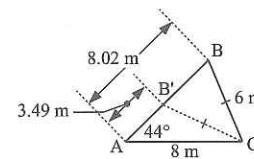
$$\therefore 36 = x^2 + 64 - 16x \cos 44^\circ$$

$$\therefore x^2 - 11.51x + 28 \approx 0$$

$$\therefore x \approx \frac{11.51 \pm \sqrt{11.51^2 - 4(1)(28)}}{2}$$

$$\therefore x \approx \frac{11.51 \pm 4.524}{2}$$

$$\therefore x \approx 8.02 \text{ or } 3.49$$



Frank needs additional information as there are two possible cases:

- (1) when  $AB \approx 8.02 \text{ m}$  and  
(2) when  $AB \approx 3.49 \text{ m}$

- b Volume = area × depth

$$\begin{aligned} &= \frac{1}{2} \times 8 \times x \times \sin 44^\circ \times 0.1 \text{ and is a maximum when } AB \approx 8.02 \text{ m} \\ &\approx 4 \times 8.02 \times \sin 44^\circ \times 0.1 \\ &\approx 2.23 \text{ m}^3 \end{aligned}$$

### REVIEW SET 11B

- 1 Using Pythagoras,

$$ED = \sqrt{6^2 + 3^2} = \sqrt{45} \text{ m}$$

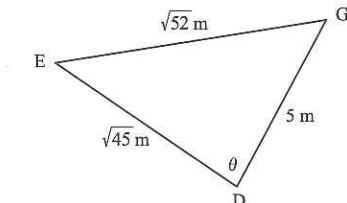
$$DG = \sqrt{4^2 + 3^2} = \sqrt{25} = 5 \text{ m}$$

$$EG = \sqrt{6^2 + 4^2} = \sqrt{52} \text{ m}$$

$$\text{Using the cosine rule, } \cos \theta = \frac{(\sqrt{45})^2 + 5^2 - (\sqrt{52})^2}{2 \times \sqrt{45} \times 5}$$

$$\therefore \theta = \cos^{-1} \left( \frac{18}{10\sqrt{45}} \right)$$

∴  $\theta \approx 74.4^\circ$  Thus  $\widehat{EDG}$  measures  $74.4^\circ$ .



$$x^2 = 8^2 + 3^2 - 2 \times 8 \times 3 \times \cos 100^\circ$$

$$\therefore x = \sqrt{8^2 + 3^2 - 48 \cos 100^\circ}$$

$$\therefore x \approx 9.0186$$

$$\begin{aligned} \text{Now } \frac{\sin \theta^\circ}{3} &\approx \frac{\sin 100^\circ}{9.0186} \\ \therefore \sin \theta^\circ &\approx \frac{3 \times \sin 100^\circ}{9.0186} \end{aligned}$$

$$\begin{aligned} \therefore \theta &= \sin^{-1} \left( \frac{3 \times \sin 100^\circ}{9.0186} \right) \\ \text{or its supplement} \\ \therefore \theta &\approx 19.1^\circ \text{ or } 160.9^\circ \end{aligned}$$

↑  
impossible

$$\therefore \theta \approx 19.1^\circ$$

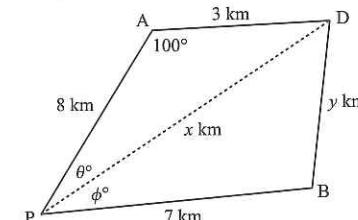
$$\therefore \phi \approx 40 - 19.1 \approx 20.9$$

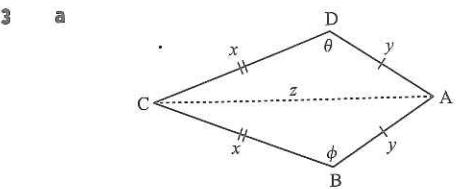
$$\therefore y^2 = x^2 + 7^2 - 2 \times x \times 7 \times \cos \phi^\circ$$

$$\therefore y \approx \sqrt{(9.0186)^2 + 7^2 - 2 \times (9.0186) \times 7 \times \cos 20.9^\circ}$$

$$\therefore y \approx 3.52$$

So, Brett has to walk 3.52 km.



Using  $\triangle ADC$ ,

$$z^2 = x^2 + y^2 - 2xy \cos \theta \quad \dots (1)$$

Using  $\triangle ABC$ ,

$$z^2 = x^2 + y^2 - 2xy \cos \phi \quad \dots (2)$$

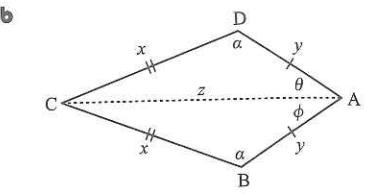
Equating (1) and (2),

$$\cos \theta = \cos \phi$$

and since  $0 < \theta, \phi < 180^\circ$ ,

$$\theta = \phi$$

$$\therefore \widehat{ADC} = \widehat{ABC}$$

Using  $\triangle DAC$ ,

$$\frac{\sin \theta}{x} = \frac{\sin \alpha}{z} \quad \dots (1)$$

Using  $\triangle BAC$ ,

$$\frac{\sin \phi}{x} = \frac{\sin \alpha}{z} \quad \dots (2)$$

$$\{\widehat{ADC} = \widehat{ABC} \text{ from a}\}$$

Equating (1) and (2),

$$\sin \theta = \sin \phi$$

$$\therefore \theta = \phi \text{ or } \theta = 180^\circ - \phi$$

but  $\widehat{DAB} = \theta + \phi < 180^\circ$ 

$$\therefore \theta = \phi$$

$$\therefore \widehat{DAC} = \widehat{BAC}$$

4 Total distance travelled =  $x + 10$  km

$$\therefore AB = (x + 10) - 4 = x + 6 \text{ km}$$

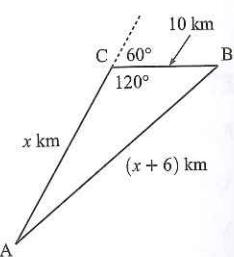
$$\text{Now } (x + 6)^2 = x^2 + 10^2 - 2 \times x \times 10 \times \cos 120^\circ$$

$$\therefore x^2 + 12x + 36 = x^2 + 100 - 20x(-\frac{1}{2})$$

$$\therefore 12x + 36 = 100 + 10x$$

$$\therefore 2x = 64$$

$$\therefore x = 32$$

 $\therefore$  the boat travelled  $x + 10 = 42$  km.

$$5 \text{ a speed} = \frac{\text{distance}}{\text{time}} \quad \therefore \text{distance} = \text{speed} \times \text{time}$$

$\therefore$  in  $t$  hours, runner A travels  $14t$  km  
and runner B travels  $12t$  km

Now  $\widehat{ASB} = 97^\circ - 25^\circ = 72^\circ$ 

$$\therefore 20^2 = (14t)^2 + (12t)^2 - 2(14t)(12t) \cos 72^\circ$$

$$\therefore 400 = 196t^2 + 144t^2 - 336t^2 \cos 72^\circ$$

$$\therefore 400 \approx 236.2t^2$$

$$\therefore t^2 \approx 1.69$$

$$\therefore t \approx 1.30 \quad \{t > 0\}$$

 $\therefore$  A and B are 20 km apart after 1 hour 18 minutes, at 2:18 pm.b When  $t \approx 1.30$ , SA  $\approx 14 \times 1.30 \approx 18.22$  kmand SB  $\approx 12 \times 1.30 \approx 15.62$  km

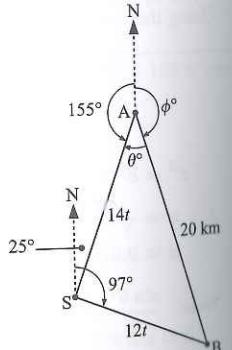
$$\therefore \cos \theta^\circ \approx \frac{18.22^2 + 20^2 - 15.62^2}{2 \times 18.22 \times 20}$$

$$\therefore \theta^\circ \approx \cos^{-1} \left( \frac{488.1}{728.8} \right)$$

$$\therefore \theta \approx 48.0$$

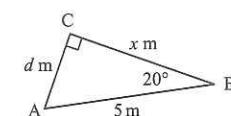
$$\therefore \phi \approx 360 - 155 - 48$$

$$\approx 157$$

 $\therefore$  B is on a bearing of  $157^\circ$  from A.

$$6 \text{ a } d^2 = x^2 + 5^2 - 2 \times x \times 5 \times \cos 20^\circ$$

$$\therefore d^2 = x^2 - (10 \cos 20^\circ)x + 25$$

c If  $\widehat{BCA}$  is a right angle, then we have

$$\text{Now } \cos 20^\circ = \frac{x}{5}$$

$$\therefore x = 5 \cos 20^\circ$$

and from b,  $d$  is minimised when  $x = 5 \cos 20^\circ$  $\therefore d$  is minimised when  $\widehat{BCA}$  is a right angle.

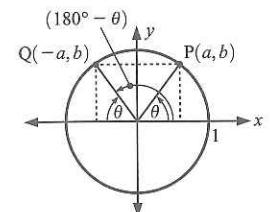
$$\text{b } d^2 \text{ is minimised when } x = \frac{-b}{2a}$$

$$\therefore x = \frac{10 \cos 20^\circ}{2}$$

$$\therefore x = 5 \cos 20^\circ$$

$$\therefore d \text{ is minimised when } x = 5 \cos 20^\circ$$

7 a



$$\therefore \cos(180^\circ - \theta) = -a$$

$$= -\cos \theta$$

b i Using  $\triangle JLM$ ,

$$x^2 = 10^2 + 15^2 - 2 \times 10 \times 15 \cos d^\circ$$

$$\therefore x^2 = 325 - 300 \cos d^\circ \quad \dots (1)$$

Using  $\triangle JLK$ ,

$$x^2 = 12^2 + 8^2 - 2 \times 12 \times 8 \cos b^\circ$$

$$\therefore x^2 = 208 - 192 \cos b^\circ \quad \dots (2)$$

Equating (1) and (2),

$$325 - 300 \cos d^\circ = 208 - 192 \cos b^\circ$$

$$\therefore 300 \cos d^\circ - 192 \cos b^\circ = 117$$

ii If  $b + d = 180$ , then  $b = 180 - d$ 

$$\therefore 300 \cos d^\circ - 192 \cos(180 - d)^\circ = 117$$

$$\therefore 300 \cos d^\circ + 192 \cos d^\circ = 117 \quad \{\text{from a}\}$$

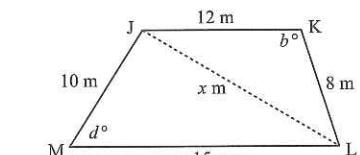
$$\therefore 492 \cos d^\circ = 117$$

$$\therefore d = \cos^{-1} \left( \frac{117}{492} \right)$$

$$\therefore d \approx 76.2$$

and  $b = 180 - d \approx 103.8$ iii If  $b + d = 180$ , then  $a + c = 180$  also  
{angles in a quadrilateral}If  $d \approx 76.2$ , then  $x \approx \sqrt{325 - 300 \cos(76.2)^\circ}$  {from (1)}

$$\therefore x \approx 15.93$$



$$\text{In } \triangle JLM, \cos \theta^\circ \approx \frac{10^2 + 15.93^2 - 15^2}{2 \times 10 \times 15.93}$$

$$\therefore \theta^\circ \approx \cos^{-1} \left( \frac{128.7}{318.5} \right)$$

$$\therefore \theta \approx 66.2$$

$$\therefore a = \theta + \phi \approx 95.4 \text{ and } c = 180 - a \approx 84.6$$

$$\text{b } y = -x^2 + 12x - 20 \text{ has } a = -1 < 0$$

its shape is , and  $y$  is maximised when  $x = \frac{-b}{2a} = \frac{-12}{2(-1)} = 6$ .

$$\text{In } \triangle JLK, \cos \phi^\circ \approx \frac{12^2 + 15.93^2 - 8^2}{2 \times 12 \times 15.93}$$

$$\therefore \phi^\circ \approx \cos^{-1} \left( \frac{333.7}{382.2} \right)$$

$$\therefore \phi \approx 29.2$$

When  $x = 6$ ,  $y = -6^2 + 12(6) - 20 = -36 + 72 - 20 = 16$

∴ the maximum value of  $y = -x^2 + 12x - 20$  is 16, which occurs when  $x = 6$ .

- b i The perimeter is 20

$$\therefore x + y + 8 = 20$$

$$\therefore y = 12 - x$$

- iii Since  $y = 12 - x$ ,  $(12 - x)^2 = x^2 + 64 - 16x \cos \theta$

$$\therefore 144 - 24x + x^2 = x^2 + 64 - 16x \cos \theta$$

$$\therefore 16x \cos \theta = 24x - 80$$

$$\therefore \cos \theta = \frac{24x - 80}{16x}$$

$$= \frac{3x - 10}{2x}$$

- c Area  $A = \frac{1}{2} \times x \times 8 \times \sin \theta$

$$= 4x \sin \theta$$

$$\therefore A^2 = 16x^2 \sin^2 \theta$$

$$= 16x^2(1 - \cos^2 \theta)$$

$$= 16x^2 \left[ 1 - \left( \frac{3x - 10}{2x} \right)^2 \right]$$

$$= 16x^2 \left[ 1 - \frac{9x^2 - 60x + 100}{4x^2} \right]$$

$$= 16x^2 - 4(9x^2 - 60x + 100)$$

$$= 16x^2 - 36x^2 + 240x - 400$$

$$= -20x^2 + 240x - 400$$

$$= 20(-x^2 + 12x - 20)$$

- 9 a  $QS^2 = 6^2 + 3^2 - 2 \times 6 \times 3 \times \cos \phi$

$$\therefore QS = \sqrt{45 - 36 \cos \phi}$$

- b i If  $\phi = 50^\circ$ ,  $QS = \sqrt{45 - 36 \cos 50^\circ} \approx 4.675$

$$\therefore \frac{\sin \theta}{7} \approx \frac{\sin 32^\circ}{4.675}$$

$$\therefore \sin \theta \approx \frac{7 \times \sin 32^\circ}{4.675}$$

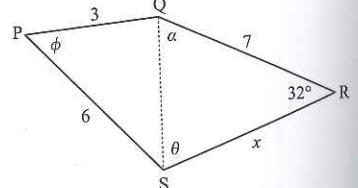
$$\therefore \theta \approx \sin^{-1} \left( \frac{7 \times \sin 32^\circ}{4.675} \right) \text{ or its supplement}$$

$$\therefore \theta \approx 52.5^\circ \text{ or } (180 - 52.5)^\circ$$

$$\therefore \widehat{R\bar{S}Q} \approx 52.5^\circ \text{ or } 127.5^\circ$$

- ii If  $\theta$  is acute, then  $\theta \approx 52.5^\circ$

$$\text{and } \alpha = 180^\circ - 32^\circ - \theta \approx 95.5^\circ$$



$$\therefore \frac{x}{\sin 95.5^\circ} \approx \frac{7}{\sin 52.5^\circ}$$

$$\therefore x \approx \frac{7 \times \sin 95.5^\circ}{\sin 52.5^\circ}$$

$$\therefore x \approx 8.78$$

$$\therefore \text{perimeter} \approx 6 + 3 + 7 + 8.78 \approx 24.8 \text{ units}$$

$$\text{c area of PQRS} = \frac{1}{2} \times 3 \times 6 \times \sin 50^\circ + \frac{1}{2} \times 7 \times 8.78 \times \sin 32^\circ \approx 23.2 \text{ units}^2$$

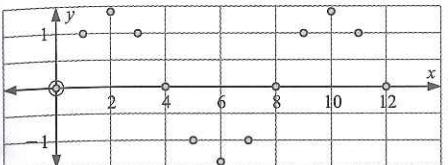
or if  $\theta$  is obtuse we can similarly calculate that the area of PQRS  $\approx 12.6 \text{ units}^2$ .

## Chapter 12

### ADVANCED TRIGONOMETRY

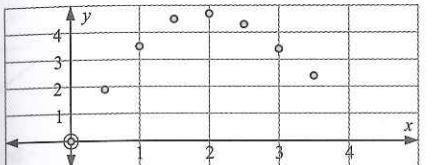
#### EXERCISE 12A

1 a



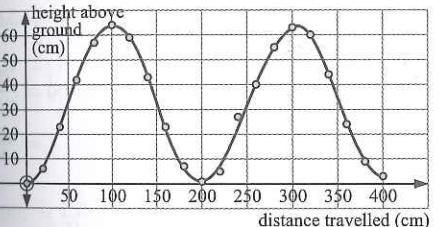
Data exhibits periodic behaviour.

c



Not enough information to say data is periodic. It may in fact be quadratic.

2 a



c A curve can be fitted to the data as the distance travelled is continuous.

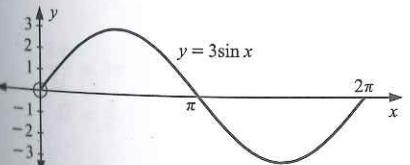
- 3 a periodic b periodic c periodic d not periodic e periodic f periodic

#### EXERCISE 12B.1

1 a

$$y = 3 \sin x$$

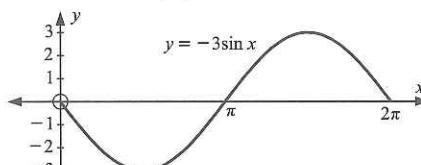
has amplitude 3 and period  $\frac{2\pi}{1} = 2\pi$   
When  $x = 0$ ,  $y = 0$ .



b

$$y = -3 \sin x$$

has amplitude  $|-3| = 3$  and period  $\frac{2\pi}{1} = 2\pi$ .  
When  $x = 0$ ,  $y = 0$ .



It is the reflection of  $y = 3 \sin x$  in the x-axis.